SPRING 1993
COMPUTER SCIENCES DEPARTMENT
UNIVERSITY OF WISCONSIN—MADISON
PH. D. QUALIFYING EXAMINATION

Modeling and Analysis
Breadth Examination

Wednesday, February 10, 1993
3:00 – 5:00 PM
113 Psychology

GENERAL INSTRUCTIONS:

1. Answer each question in a separate book.
2. Indicate on the cover of *each* book the area of the exam, your code number, and the question answered in that book. On *one* of your books list the numbers of *all* the questions answered. *Do not write your name on any answer book.*
3. Return all answer books in the folder provided. Additional answer books are available if needed.

SPECIFIC INSTRUCTIONS:

Answer any *four* of the following *five* questions.

POLICY ON MISPRINTS AND AMBIGUITIES:

The Exam Committee tires to proofread the exam as carefully as possible. Nevertheless, the exam sometimes contains misprints and ambiguities. If you are convinced a problem has been stated incorrectly, mention this to the proctor. If necessary, the proctor can contact a representative of the area to resolve problems during the *first hour* of the exam. In any case, you should indicate your interpretation of the problem in your written answer. Your interpretation should be such that the problem is nontrivial.

MA Breadth Screening Exam

Spring 1993

Directions: Answer **FOUR** of the **FIVE** questions below. Show all your work.

1. Short Answer Questions:

   (a) What is the relationship between the mean response times, R_i and R_{ii}, for systems (i) and (ii) below?

   (i) An M/M/1 system with arrival rate λ and mean service time S.

   (ii) An M/M/1 system with arrival rate 2λ and mean service time $\frac{S}{2}$.

   (b) Which random variable has a higher coefficient of variation?

   (i) X, an exponentially distributed r.v. with parameter μ, or

   (ii) Y, where Y = X + 1

   (c) Consider the discrete-parameter Markov chain shown in the Figure below, where each state transition is labeled with the probability of making the transition at the end of the time step, conditioned on being in the given state in the current time step. What is the steady state probability that the system is in state 0?

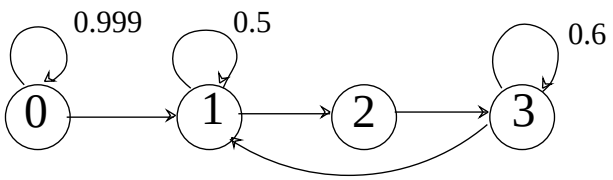

Figure 1:

2. Suppose we have a system A that is accurately modeled by a product-form queueing network. Now suppose we upgrade the system so that the mean service time at every resource is reduced by 50%. Call the upgraded system B and assume it is also accurately modeled by the product-form queueing network with the corresponding changes in service time parameters.

   Use either the product-form equations for the network state probabilities or the mean value analysis (mva) equations for the network to explain what happens quantitatively to the resource utilizations, mean queue length at each resource, network throughput, and network mean response time when the system is upgraded, for each of the following cases:

   (a) The queueing network is a closed, single-class network.

   (b) The queueing network is an open, single-class network.

3. Consider the following M/M/1 FCFS queue with impatient customers. Customers arrive with rate λ and their service demand is exponentially distributed with parameter μ. If an arriving customer does not begin service before an exponentially distributed amount of time with parameter α, the customer simply leaves the queue. (Hint: Consider the rate at which each customer impatiently departs the queue.)

   (a) Draw the Markov Chain for the system.

   (b) Under what conditions is this queue stable?

4. (a) When is a simulation functional model inconsistent?

   (b) When is a simulation functional model well described?

   (c) How would you use a well described simulation functional model to predict the behavior of a system that is driven by a stochastic process?

5. A number of transactions access pages from a common buffer pool that contains a total of 100 pages. Each transaction executes on its own server for a random amount of time with mean 10 milliseconds and then makes a request for a page. If no other transaction is accessing the page, the transaction holds the page for an average of 200 milliseconds. Otherwise the transaction waits in a FCFS queue for the page. After completing its page access the transaction releases the page and returns to its processing phase. Suppose that 80% of the total page requests are uniformly spread across pages 1 through 20 (i.e., the *hot* pages) and the remaining 20% of the requests are uniformly distributed across all 100 pages.

   (a) Draw the queueing network model of this system.

   (b) What is the maximum possible throughput of the system?