CS810: Homework 1 Due date: Thursday , Feb 13th, 2003

1. Given a directed graph G, and two vertices s and t. Show that there is no path from s to t, iff there is a partition of the vertex set of G (called a cut) separating s and t for which there is no edge from the part containing s to that part containing t.

2. Show that 2-SAT is in P.

3. In class we gave a proof that for any NP-complete decision problem Π, the functional search problem (i.e., finding a solution when there is one, e.g., finding a Hamiltonian Circuit when one exists), to the decision problem (i.e., a black box answering yes/no queries.) We ddid this in particular for Hamiltonian Circuit problem.

   Write out this proof carefully, first for the SAT problem, then for the Hamiltonian Circuit problem.

   Then for the Hamiltonian Circuit problem, give a more direct proof.

4. An n-dimensional grid is a graph $G = (V, E)$ where

   $$V = \{(i_1, i_2, \ldots, i_n) \mid 1 \leq i_j \leq m_j, 1 \leq j \leq n\}$$

   and $E = \{(v_1, v_2) \mid \ v_1$ and v_2 differ in only one coordinate and by exactly 1.$\}$.

   For what values of m_j and n does G have a Hamiltonian Circuit?

   Let G be the graph whose vertices are the squares of an 8×8 chess board and whose edges are the legal moves of a knight (ie one square this way and 2 squares that way.) Find a Hamiltonian Circuit for G.

Note:

Please be concise.