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A PARTICLE MODEL OF OCEAN WAVES GENERATED BY EARTHQUAKES

by

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Abstract

In this paper a new particle approach to the study of fluid dynamics is applied to the generation of ocean waves by earthquakes. Major interest is centered on a molecular type compression wave as the key mechanism for producing surface water waves. Computer examples for three basic oceanic topographies are studied, each yielding the breaking of ocean waves into a bay area.

1. Introduction

Phenomena associated with large scale geophysical phenomena require time and/or physical scales of such large orders of magnitude that they are exceptionally difficult to model (see, e.g., references [1]-[3] and the numerous additional references contained therein). Such phenomena are usually studied from a gross, continuum point of view, with the equations of the more sophisticated models now being solved numerically on high speed digital computers (see, e.g., [4]).

In this paper we will introduce a discrete, noncontinuum type model to help in the understanding of such large scale phenomena. These models have already proved to be successful in revealing the mechanisms of nonlinear physical phenomena related to fluid flow, heat transfer, and elastic vibration [5]. In the present paper we will concentrate on ocean waves which are generated by earthquakes, with one ultimate aim being the understanding of tsunami (tidal wave) generation and flow [6]. The rationale behind the model is as follows. Oceanic fluid motions are determined from initial data largely by the classical molecular interaction forces and by gravity. Qualitatively, molecules attract when they are far apart, repel when they are close, and repulsion is a much greater force than is attraction [7]. Moreover, intermolecular forces are local. In the model to be described, we will use relatively large and relatively few fluid units called particles, in place of molecules; we will rescale the intermolecular force parameters appropriately; and we will limit the force range to relatively short distances. A completely analogous approach to the study of shock waves was first recommended by von Neumann [8].

2. Basic Definitions and Formulas

For clarity, let us first give precise statements of the basic mathematical definitions and formulas which will be needed.

For positive time step Δt, let $t_k = k\Delta t$, $k = 0,1,2,\ldots$. For $i = 1,2,\ldots,N$, let particle P_i have mass m_i and at time t_k let P_i be located at $\vec{r}_{i,k} = (x_{i,k}, y_{i,k})$, have velocity $\vec{v}_{i,k} = (v_{i,k,x}, v_{i,k,y})$ and have acceleration $\vec{a}_{i,k} = (a_{i,k,x}, a_{i,k,y})$. Let position, velocity and acceleration be related by the "leap-frog" formulas ([5], p. 107):

$$(2.1)\qquad \vec{v}_{i,\frac{1}{2}} = \vec{v}_{i,0} + \frac{\Delta t}{2}\vec{a}_{i,0}$$

$$(2.2)\qquad \vec{v}_{i,k+\frac{1}{2}} = \vec{v}_{i,k-\frac{1}{2}} + (\Delta t)\vec{a}_{i,k}, \quad k = 1,2,\ldots$$

$$(2.3)\qquad \vec{r}_{i,k+1} = \vec{r}_{i,k} + (\Delta t)\vec{v}_{i,k+\frac{1}{2}}, \quad k = 0,1,2\ldots .$$

If $\vec{F}_{i,k}$ is the force acting on P_i at time t_k, where $\vec{F}_{i,k} = (F_{i,k,x}, F_{i,k,y})$, then we assume that force and acceleration are related by

$$(2.4)\qquad \vec{F}_{i,k} = m_i\vec{a}_{i,k}.$$

Once an exact structure is given to $\vec{F}_{i,k}$, the motion of each particle will be determined recursively and explicitly by (2.1)-(2.4) from prescribed initial data. The special structure to be used is described as follows.

At time t_k, let $r_{ij,k}$ be the distance between P_i and P_j. Let G^* (coefficient of attraction), H^* (coefficient of repulsion),

β (exponent of attraction) and α (exponent of repulsion) be determined by P_i and P_j subject to the constraints $G^* \geq 0$, $H^* \geq 0$, $\alpha \geq \beta$ (see [9]). Then the force $(\bar{F}_{i,k,x}, \bar{F}_{i,k,y})$ exerted on P_i by P_j is given by

$$(2.5) \qquad \bar{F}_{i,k,x} = \left[\frac{-G^* m_i\ m_j}{r_{ij,k}^{\beta}} + \frac{H^* m_i\ m_j}{r_{ij,k}^{\alpha}} \right] \frac{x_{i,k} - x_{j,k}}{r_{ij,k}}$$

$$(2.6) \qquad \bar{F}_{i,k,y} = \left[\frac{-G^* m_i\ m_j}{r_{ij,k}^{\beta}} + \frac{H^* m_i\ m_j}{r_{ij,k}^{\alpha}} \right] \frac{y_{i,k} - y_{j,k}}{r_{ij,k}} .$$

The total force $(F^*_{i,k,x}, F^*_{i,k,y})$ on P_i due to all the other $N-1$ particles is given by

$$(2.7) \qquad F^*_{i,k,x} = \sum_{\substack{j=1 \\ j \neq i}}^{N} \bar{F}_{i,k,x}; \quad F^*_{i,k,y} = \sum_{\substack{j=1 \\ j \neq i}}^{N} \bar{F}_{i,k,y} .$$

Finally, we include gravity into the model by

$$(2.8) \qquad F_{i,k,x} = F^*_{i,k,x}; \quad F_{i,k,y} = F^*_{i,k,y} - 980\ m_i .$$

The formulation (2.1)-(2.8) is explicit and economical, though nonconservative. Conservation of energy and momenta can be achieved [5], but only through an implicit, less economical approach.

3. Computer Generation of Ocean-Bay Configurations

We will consider three ocean-bay geometries, called geometries A, B and C, as shown in Figure 3.1. The (x,y) coordinates of the points are A(-4,2.3), B(-4,2), C(0,2), D(0,0), E(4,0), F(4,2.3), G(0,1.7293), H(1.0444,0), I(-4,2.4), J(-2,2), K(2,0), L(4,2.4). (The reason for the complexity of the coordinates of G and H will become apparent shortly.) Our intent is to perturb the ocean floors in each of geometries A, B and C and to observe then the wave motions which result. Before one can make waves, however, one must have water. So, our problem in this section is to simulate the bay and ocean water, and this will be done as follows.

Consider first geometry A. For $N = 221$, let particles P_i, $i = 1,2,\ldots,N$, each have zero velocity and be distributed in a uniform, triangular pattern [5, p 75] throughout the bay and ocean areas. For clarity assume $m_i \equiv 100$, $H^* = G^* = 1$, $\alpha = 3$, and $\beta = 1$ in (2.5)-(2.6). To assure that forces are short range, let each interparticle force be zero when the distance between two particles is greater than 0.25. To achieve physical equilibrium, we now merely let the particles interact according to (2.1)-(2.8) until a stable configuration results ([10], [11]), since there is no analytical method for resolving this complex N-body problem.

There remains, however, a final consideration before the above calculations can be executed. We will assume that particles are reflected symmetrically, but with strongly damped velocities, when colliding with a wall. The damping factor will always be 0.1 at any vertical

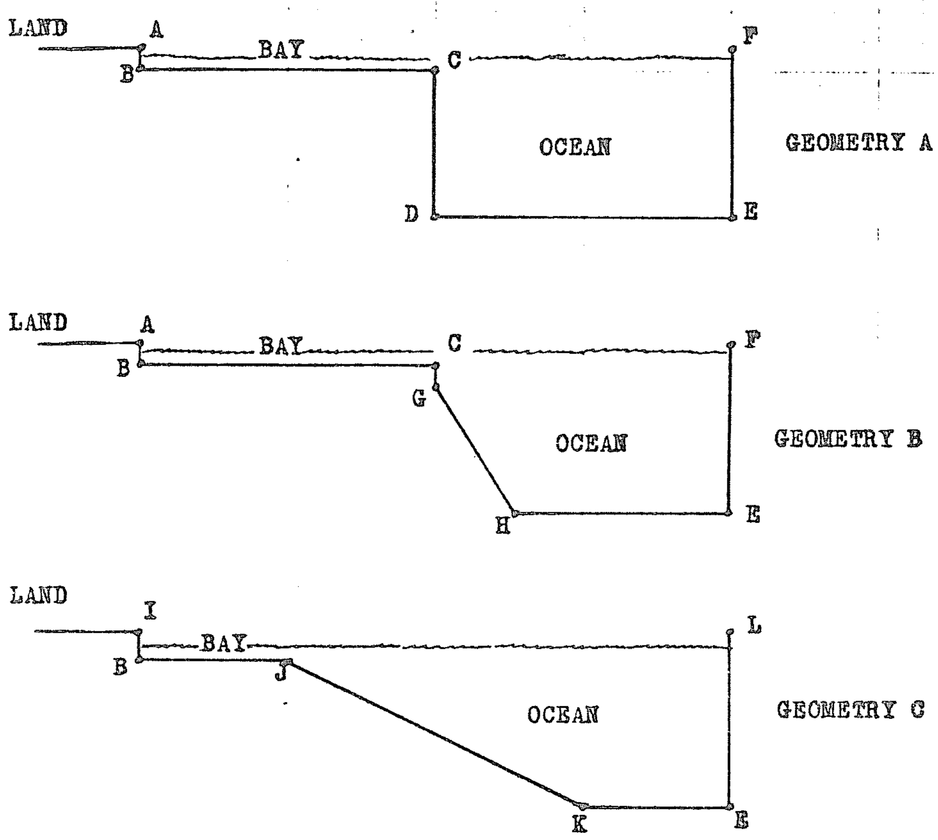


FIGURE 3.1

or horizontal wall in any of the three geometries. With this modification, the particles were allowed to interact with $\Delta t = 10^{-4}$ in accordance with (2.1)-(2.8). At first the interaction was strong, but within three seconds the system kinetic energy peaked and thereafter decreased in an oscillatory fashion. After nine seconds, the system kinetic energy oscillated, but its minimum no longer decreased and the resulting configuration was taken to be an equilibrium state. This configuration is shown in Figure 3.2 and the exact positions and velocities are given in Table 1. The basic FORTRAN program, which required only minor modifications for all the examples to be described, is given in the Appendix of [12].

If one next draws the segment joining the two particles with centers G(0, 1.7293) and H(1.0444, 0) in Figure 3.2 and eliminates the particles below this segment, then one has immediately the stable configuration for geometry B, which is shown in Figure 3.3. This configuration has 200 particles.

Finally, if one draws in Figure 3.2 the line segment joining the points J(-2,2) and K(2,0) , eliminates those particles which lie below this segment, and inserts new particles, by symmetric reflection about the Y axis, into the void between $x = -2$ and $x = 0$, then merely allowing continued particle interaction for 2.1 seconds yields a stable geometry C configuration as shown in Figure 3.4. The exact positions and velocities of the 214 particles in the figure are given in Table 2. It is important to note, however, that, for simplicity, particle collision with JK was treated as follows. Whenever a particle's

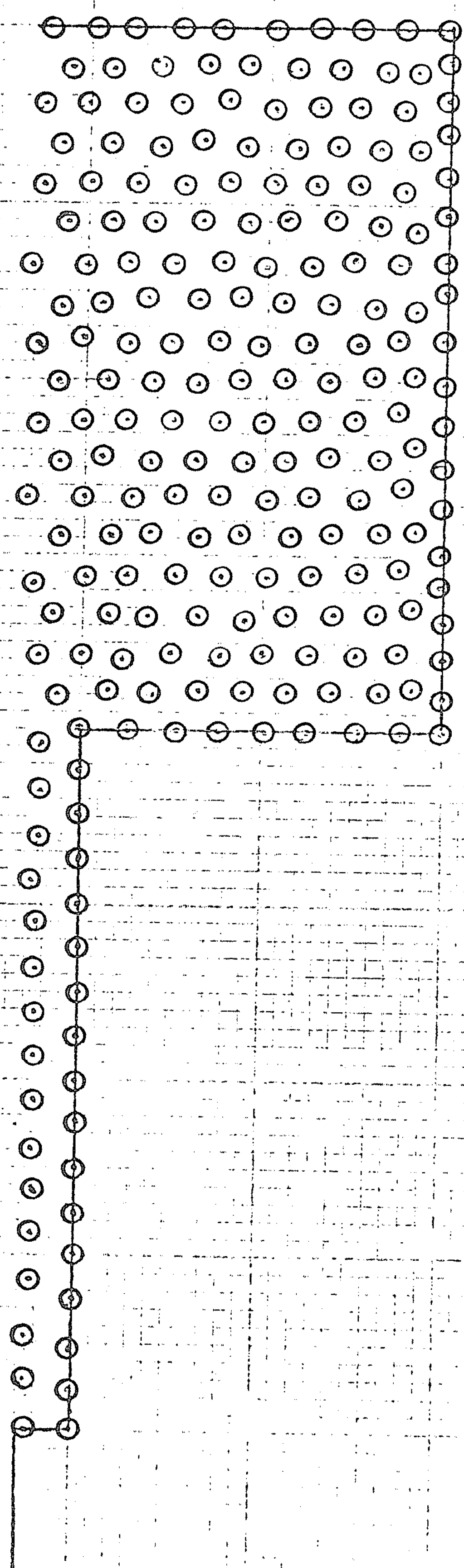


FIGURE 3.2

TABLE 1

x	y	v_x	v_y	x	y	v_x	v_y
-3.9927	2.2578	-4.3952	-3.8075	0.4382	1.2294	-2.4551	-6.1267
-3.7105	2.2690	0.9354	-3.2726	0.4440	1.4817	2.1407	-2.3745
-3.4580	2.2650	2.8755	3.9988	0.4441	1.7306	0.3134	1.9987
-3.1227	2.2455	1.3512	2.2408	0.4422	1.9824	3.3195	-0.2656
-2.8807	2.2270	4.1389	-0.2405	0.4323	2.2421	0.3592	-0.5651
-2.6311	2.2196	4.6376	-0.9073	0.6053	0.0001	0.0569	-0.7888
-2.3900	2.2212	-3.1164	6.7971	0.7004	0.1920	-4.3656	-2.3999
-2.1345	2.2277	-0.3421	5.5138	0.6684	0.3860	1.5375	-1.7353
-1.8757	2.2174	0.9384	0.2843	0.6650	0.6092	-3.4393	1.9123
-1.6255	2.2195	2.6711	0.4874	0.6630	0.8555	4.7631	-1.8689
-1.3665	2.2314	1.2481	-0.7916	0.6585	1.1092	1.1905	1.2395
-1.1105	2.2251	0.5594	-0.2571	0.6676	1.3583	1.2372	-0.9721
-0.8596	2.2847	0.7739	1.2304	0.6642	1.6098	-0.6567	-2.0998
-0.6183	2.2253	-2.0775	0.9177	0.6734	1.8665	1.7356	-6.8195
-0.3523	2.2409	0.7575	-1.4317	0.6687	2.1168	4.3367	-1.9121
-0.1021	2.2112	0.4242	-0.9680	0.8393	0.0000	0.0047	0.0613
-4.0000	2.0000	0.0059	0.0010	0.9245	0.2382	-2.9468	2.2737
-3.7500	2.0000	0.0606	0.0178	0.8976	0.4855	1.9197	0.1827
-3.5000	2.0000	0.0054	0.0089	0.8845	0.7320	3.1437	4.0884
-3.2500	2.0000	-0.0654	0.0089	0.8938	0.9793	-3.2822	-1.2700
-3.0000	2.0000	0.0053	0.0089	0.8843	1.2214	0.9059	-3.1274
-2.7500	2.0000	0.0229	0.0572	0.8913	1.4711	4.5934	1.2663
-2.4999	2.0000	0.5688	-0.1392	0.8827	1.7312	2.7125	0.2942
-2.2499	2.0000	0.0000	0.0089	0.8920	1.9821	4.7395	-3.2951
-1.9993	2.0001	-0.0333	0.0572	0.8510	2.2875	-0.7592	-2.0623
-1.7494	2.0000	0.0000	0.0089	1.0444	0.0001	-0.0024	0.0808
-1.4993	2.0000	-0.0000	0.0089	1.1649	0.1790	4.3000	0.5862
-1.2492	2.0000	-0.0000	0.0089	1.1262	0.3782	3.0289	-2.1217
-0.9992	2.0000	-0.0000	0.0089	1.1093	0.6132	-5.5256	-1.6277
-0.7490	2.0000	0.0000	0.0089	1.1055	0.8625	-1.2328	-2.1212
-0.4989	2.0000	0.0000	0.0089	1.1076	1.1057	2.1661	0.9715
-0.2489	2.0000	0.0000	0.0089	1.1145	1.3569	4.6380	-3.4542
0.0007	1.9797	0.2283	-0.6282	1.1092	1.6125	-1.4644	-4.5414
0.0002	1.7293	0.0416	-0.8196	1.1075	1.8657	-2.7490	1.8826
0.0000	1.4795	0.0417	-0.2926	1.1004	2.1373	-2.4742	-5.4040
0.0002	1.2293	0.0973	-0.6316	1.2944	0.0000	0.0089	0.0737
0.0001	0.9793	0.0164	0.4435	1.3785	0.2356	-0.5104	-3.2738
0.0008	0.7295	-0.8132	-0.8296	1.3330	0.4900	-2.3942	-2.0983
0.0000	0.4857	0.0446	0.0144	1.3285	0.7351	6.0673	-4.7671
0.0000	0.2438	0.0548	0.0046	1.3263	0.9850	3.6562	0.6475
0.0001	0.0000	0.0132	0.0074	1.3373	1.2393	0.9006	2.4843
0.1932	0.0001	-0.0014	0.1368	1.3424	1.4929	2.0797	-4.2756
0.2124	2.1149	4.0860	-6.8381	1.3370	1.7509	-1.3225	-0.7286
0.2392	0.1827	-1.4226	3.2393	1.3213	2.0066	4.6558	2.8379
0.2257	0.3772	4.4933	4.3047	1.3491	2.3192	5.3025	-5.8719
0.2224	0.6169	-2.0559	4.0336	1.5528	2.1276	-4.2833	0.2055
0.2241	0.8562	-3.9304	4.1129	1.5041	0.0001	0.0452	0.0917
0.2215	1.1008	0.5657	2.7400	1.5137	0.1765	-4.1326	5.6353
0.2273	1.3539	1.6217	-1.0550	1.5680	0.3846	-6.2516	0.3399
0.2176	1.6051	2.2364	0.1770	1.5495	0.6182	-4.7335	2.4189
0.2184	1.8555	-1.5040	-0.3102	1.5450	0.8700	-2.9258	5.1179
0.4060	0.0001	0.0148	0.2395	1.5485	1.1212	-5.0466	0.4761
0.4610	0.2444	-2.3539	6.7116	1.5610	1.3746	-1.4559	1.9257
0.4401	0.4878	2.1325	0.7822	1.5531	1.6254	1.5331	-0.7394
0.4460	0.7309	6.1484	0.7533	1.5565	1.8755	-0.4951	-5.2762
0.4431	0.9805	0.7638	3.3098	1.7576	2.2654	-2.4935	2.6057

TABLE 1 (continued)

x	y	v_x	v_y	x	y	v_x	v_y
1.7540	0.0000	-0.0381	0.0685	2.8937	1.5672	-0.8135	3.3353
1.8117	0.2570	-2.9486	-0.7436	2.8985	1.6123	1.0411	-2.0304
1.7753	0.5036	-1.9141	0.1042	2.8934	1.8620	-1.1616	-1.3040
1.7629	0.7511	-3.1982	4.8849	3.1300	2.2532	-4.7266	-0.9443
1.7634	1.0029	-2.6679	-0.4006	3.1468	0.0001	0.1049	-1.0541
1.7847	1.2527	3.3426	1.7053	3.0756	0.2367	4.4153	-4.2313
1.7761	1.5041	1.2196	0.4369	3.0986	0.4892	3.842[illegible]	-5.0259
1.7774	1.7595	9.1355	-5.4691	3.1100	0.7382	4.1693	0.1172
1.7699	2.0036	-4.4357	-1.3027	3.1139	0.9884	0.3490	-2.9901
1.9716	0.0000	0.0389	0.1217	3.1202	1.2356	0.3136	-2.2353
2.0461	0.1763	0.2771	5.8094	3.1165	1.4854	-1.1476	-0.2398
2.0025	0.3940	5.4335	-0.1753	3.1172	1.7366	3.7142	1.9000
1.9870	0.6392	-4.5376	-0.2498	3.1237	1.9880	-8.5495	-0.5698
1.9901	0.8821	-1.8785	0.8633	3.2976	0.1820	2.0624	1.3741
2.0072	1.1335	-2.6837	2.6964	3.3265	0.4000	0.0591	2.8430
1.9840	2.1422	4.1369	-3.7960	3.3239	0.6150	0.7090	1.6714
1.9924	1.5783	-0.0966	-1.7796	3.3308	0.8610	-7.2551	0.6830
1.9949	1.6283	-5.6800	-4.2242	3.3349	1.1067	1.4519	1.0948
1.9945	1.8918	-2.8746	-0.3086	3.3406	1.3627	6.5473	6.5465
2.2240	2.0367	3.3708	-1.8449	3.3362	1.6130	-1.6262	0.1140
2.2216	0.0000	-0.0137	0.0479	3.3401	1.8640	2.1177	-0.3736
2.2225	0.2877	3.5582	0.3998	3.3890	0.0001	0.0017	0.1011
2.2004	2.2888	0.4539	-3.5728	3.3449	2.1158	1.1877	-5.8372
2.2041	0.5191	-4.0409	-1.0692	3.5604	1.9898	0.2447	-0.3629
2.2194	0.7582	-5.8259	-7.4251	3.5909	0.0000	-0.0226	0.0498
2.2191	0.9963	4.7182	0.2963	3.5233	0.2460	0.9128	2.7140
2.2310	1.2444	2.0605	-2.8394	3.5581	0.4922	4.8991	-6.9419
2.2180	1.4923	3.144[illegible]	-0.5657	3.5535	0.7304	1.5995	1.8559
2.2258	1.7437	7.2543	1.2344	3.5489	0.9814	-1.3606	-2.4897
2.4369	2.1498	0.7762	2.3575	3.5624	1.2320	0.1152	-1.1047
2.6740	2.3003	-4.4353	3.2963	3.5634	1.4817	0.0484	-0.5201
2.4715	0.0000	0.0184	0.1239	3.5591	1.7313	-2.4338	-1.9520
2.4015	0.1793	3.1908	4.7155	3.7760	2.1190	-2.4959	1.6066
2.4367	0.4090	-5.5990	-2.9656	3.5679	2.2534	3.1357	-1.4252
2.4365	0.6373	-1.5348	6.8849	3.8353	0.0001	0.0001	0.1376
2.4395	0.8838	6.6343	-2.6294	3.7571	0.1865	-0.0152	-0.1992
2.4537	1.1200	2.6347	0.6360	3.7763	0.3789	-3.0058	0.7062
2.4539	1.3690	2.6244	-1.3678	3.7700	0.6121	3.1292	-0.4559
2.4510	1.6274	0.3125	3.5076	3.7931	0.8589	-2.0024	-3.7922
2.4359	1.8752	6.2494	-0.2063	3.7919	1.1079	3.8072	-5.1020
2.6654	2.0237	-1.1838	-2.6471	3.7809	1.3591	1.7223	-2.6517
2.6884	0.0000	-0.1012	-0.5177	3.7822	1.6061	-4.5538	-1.0345
2.6257	0.2626	-0.0346	-1.1540	3.7895	1.8607	0.6093	0.9935
2.6668	0.5072	1.6478	-2.4658	3.9999	0.0000	-0.0129	0.0073
2.6557	0.7548	5.2852	1.5145	4.0000	0.2445	-0.0533	0.0070
2.6681	0.9986	1.6708	2.5410	4.0000	0.4915	-0.0483	-0.1240
2.6700	1.2433	0.0453	-5.7787	3.9996	0.7326	-0.0506	0.6370
2.6695	1.4936	-1.8715	3.4556	4.0000	0.9819	-0.0432	0.0337
2.6619	1.7627	1.7812	-6.4511	3.9999	1.2317	-0.0387	-0.0641
2.6950	2.1190	-6.7573	0.4893	3.9999	1.4816	-0.0457	0.6501
2.9383	0.0000	0.0007	0.0867	4.0000	1.7313	-0.0475	0.0297
2.8356	0.1887	-1.1535	-1.5314	3.9999	1.9818	-0.0421	0.0826
2.8696	0.3885	2.1432	-5.0039	3.9999	2.2320	-0.0437	-0.2821
2.9000	0.6108	2.9000	-4.9153				
2.8930	0.8635	4.5058	1.0217				
2.8929	1.1175	1.6700	5.6658				

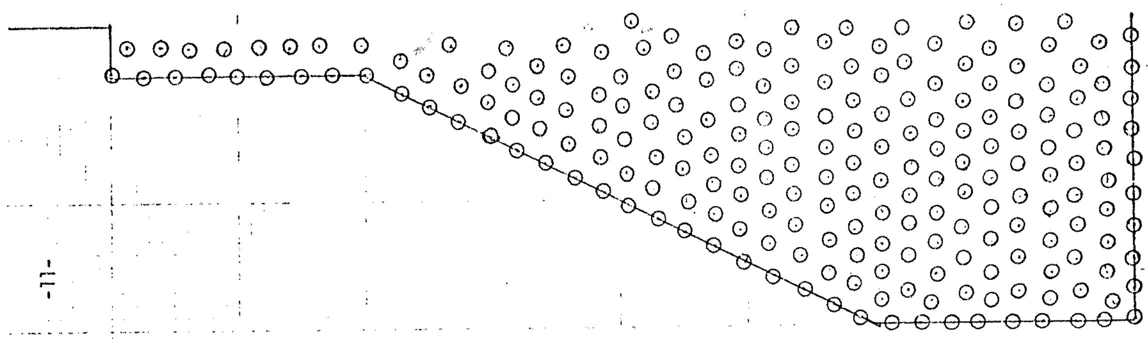


FIGURE 3.4

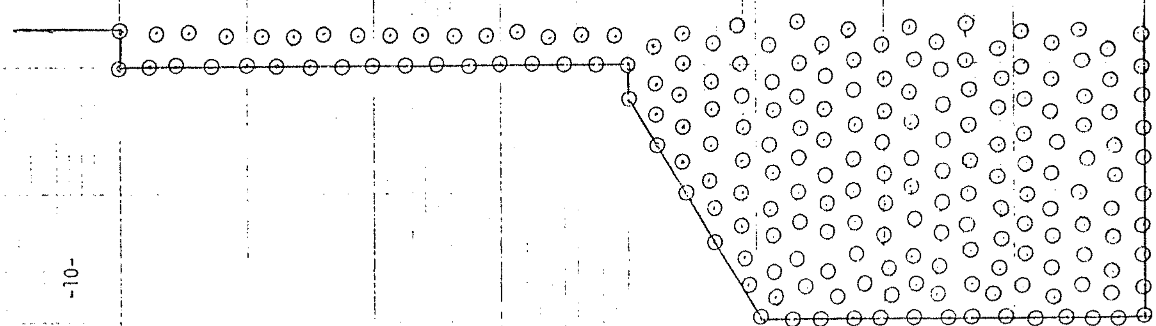


FIGURE 3.3

TABLE 2

x	y	v_x	v_y	x	y	v_x	v_y
-3.8744	2.2168	1.5655	0.9778	0.2586	1.8573	11.8639	1.3394
-3.6243	2.2168	0.2396	-0.1159	0.5339	0.7331	0.	0.
-3.3711	2.2175	-1.1637	-1.1019	0.5210	0.9843	-1.5906	-9.3356
-3.1186	2.2176	0.1884	-0.2931	0.5104	1.2306	5.1514	-0.9017
-2.8671	2.2186	0.2463	-0.6771	0.5072	1.4746	4.3350	-0.6142
-2.6148	2.2169	-0.1496	0.8838	0.4927	1.7675	0.3598	-12.6040
-2.3626	2.2170	-1.4705	-1.1715	0.4625	2.0234	2.2397	-1.6354
-2.0669	2.1832	-0.0040	-0.1796	0.4288	2.2877	3.1260	3.4047
-4.0000	2.0000	0.0027	-0.1011	0.7572	0.6214	0.	0.
-3.7497	2.0000	-0.0000	0.0089	0.7436	0.8783	-7.6137	-0.1992
-3.4955	2.0000	0.0000	0.0089	0.7360	1.1232	4.2368	12.8063
-3.2447	2.0000	-0.0000	0.0178	0.7268	1.3639	5.0762	3.4455
-2.9917	2.0000	0.0000	-0.0802	0.6980	1.6285	0.3428	-0.1734
-2.7416	2.0000	-0.0000	0.0089	0.7104	1.9306	-0.8460	-5.4521
-2.4878	2.0000	-0.0006	-0.0712	0.6719	2.1816	-6.3617	2.5066
-2.2375	2.0000	0.0000	0.0089	0.9809	0.5095	0.	0.
-1.9380	1.9690	0.	0.	0.9554	0.7596	-1.0870	3.4076
-1.7409	2.1320	-0.7554	-1.2847	0.9602	1.0066	3.0659	5.9188
-1.7144	1.8572	0.	0.	0.9472	1.2602	-0.9134	6.2179
-1.5022	2.0110	2.8962	-1.4862	0.9306	1.5108	-4.4046	2.5105
-1.4907	1.7454	0.	0.	0.9406	1.7544	1.5297	-7.3556
-1.3171	2.1723	-2.2184	-5.4834	0.9484	2.0250	-4.7860	3.2906
-1.2749	1.9050	6.5834	0.0601	0.9248	2.2788	0.5843	1.3674
-1.2658	1.6329	0.	0.	1.2044	0.3978	0.	0.
-1.0811	2.0432	4.1787	-0.2404	1.1804	0.6437	-0.7216	7.2397
-1.0581	1.7749	4.2742	-3.6220	1.1709	0.8970	2.0603	1.1659
-1.0367	1.5183	0.	0.	1.1676	1.1435	4.8609	5.3631
-0.8911	2.1971	5.7693	-0.6421	1.1633	1.3894	-8.0794	-3.7853
-0.8494	1.9137	-0.8817	-6.9642	1.1622	1.6333	2.5189	-4.0373
-0.8304	1.6695	0.4057	6.5023	1.1633	1.8997	1.8515	-0.6304
-0.8109	1.4054	0.	0.	1.1681	2.1523	1.5621	1.3255
-0.6508	2.0888	1.6847	0.7515	1.4274	0.2863	0.	0.
-0.6195	1.8073	3.9642	-3.6922	1.4085	0.5277	-0.2239	-7.8833
-0.6129	1.5565	-1.9491	3.8403	1.3956	0.7765	7.8218	5.8804
-0.5852	1.2926	0.	0.	1.3958	1.0113	-4.2190	4.2585
-0.4505	2.2410	2.4542	-5.3888	1.3889	1.2655	-2.1621	1.1954
-0.3994	1.9807	4.2690	-1.3940	1.3781	1.5201	-0.4060	0.4732
-0.4021	1.7073	-0.3003	-2.8939	1.3854	1.7645	-4.9855	-1.7238
-0.3734	1.4379	0.9672	-5.8044	1.3967	2.0449	0.9226	2.3066
-0.3610	1.1805	0.	0.	1.3575	2.3193	-9.0897	-0.5641
-0.1617	2.1572	0.6368	10.1560	1.6200	2.1772	0.2294	3.7333
-0.1888	1.8370	-0.8231	-1.7445	1.6510	0.1745	0.	0.
-0.1634	1.5847	-2.4518	-9.1115	1.6268	0.4169	4.2369	2.5434
-0.1523	1.3213	0.6460	3.3613	1.6066	0.6535	8.5262	-2.3235
-0.1371	1.0686	0.	0.	1.6213	0.9019	2.3934	3.8628
0.1009	2.3936	-1.7376	0.6392	1.6069	1.1458	6.1658	-0.8622
0.0222	1.9824	-8.1873	3.0182	1.6005	1.3960	-4.3168	0.3848
0.0394	1.7321	-2.3202	2.5104	1.6119	1.6556	0.0112	-5.2754
0.0495	1.4680	-5.7810	5.2909	1.6125	1.9010	-0.3023	2.9074
0.0676	1.2063	-6.9530	0.3151	1.8351	2.3212	-3.5971	-0.4845
0.0866	0.9567	0.	0.	1.8411	0.3027	7.1245	1.6939
0.1981	2.1622	4.7014	4.0654	1.8385	0.5335	5.5045	-1.1892
0.3102	0.8449	0.	0.	1.8303	0.7632	-2.7145	-3.8622
0.2974	1.0980	-1.0504	3.7531	1.8300	1.0231	-0.2322	-6.2823
0.2820	1.3342	0.1434	6.6941	1.8254	1.2707	9.2582	-14.2435
0.2862	1.5891	-4.4952	8.3552	1.8238	1.5152	0.6446	-9.9227

TABLE 2 (continued)

x	y	v_x	v_y
1.8284	1.7780	-2.6659	-5.7797
1.8526	2.0346	3.7290	-4.8043
1.8746	0.0627	0.	0.
2.0583	0.1917	4.2065	-9.7599
2.0520	0.4236	1.1454	3.7729
2.0475	0.6477	-1.2090	3.8515
2.0450	0.8931	4.2635	3.9054
2.0472	1.1434	3.5746	1.2251
2.2768	2.3296	4.4646	3.4804
2.0492	1.3948	4.1386	5.7631
2.0515	1.6590	-1.8424	1.4905
2.0659	1.9047	-4.7564	10.6438
2.2719	2.0578	-6.1631	-8.8904
2.1161	0.0000	0.0042	0.1253
2.2442	0.2708	4.1846	5.8509
2.0584	2.1777	1.8926	4.5380
2.2593	0.5236	2.1172	1.0692
2.2659	0.7656	6.1788	3.8250
2.2672	1.0105	0.6256	-0.7608
2.2731	1.2559	-0.1382	-1.4768
2.2747	1.5070	3.0692	7.7426
2.2707	1.7752	-4.6827	-2.7494
2.5026	2.2181	10.8726	5.5587
2.7201	2.3813	1.1130	-0.0344
2.3366	0.0000	-0.0226	0.0855
2.4449	0.1845	-0.3658	2.1151
2.4660	0.3895	6.3058	-3.7198
2.4824	0.6317	-5.5085	-1.3249
2.4804	0.8811	-6.6078	-7.5343
2.4962	1.1325	0.2978	0.9170
2.4895	1.3817	2.4262	-3.9943
2.4955	1.6259	-6.6519	-4.6809
2.4808	1.8987	2.2834	1.5888
2.6951	2.0265	3.9493	0.5369
2.5866	0.0000	0.0119	0.0632
2.6843	0.2385	1.1353	-2.6621
2.6941	0.5006	-5.1399	-0.5434
2.6889	0.7559	3.5775	-4.3874
2.6951	1.0014	-1.4914	-2.2355
2.7055	1.2534	4.2322	-3.2276
2.7079	1.5129	3.8123	-1.2703
2.7023	1.7516	4.5356	2.0724
2.8998	2.1638	-1.2333	7.9913
2.8025	0.0001	-0.0255	0.1072
2.9020	0.1708	0.5128	3.2861
2.9052	0.3847	3.8820	3.1636
2.9050	0.6326	-2.7957	-0.1420
2.9034	0.8738	1.8291	-3.8421
2.9083	1.1260	2.7736	2.8009
2.9139	1.3743	-10.8324	-4.1366
2.9272	1.6349	-13.7833	-1.3770
2.9249	1.8961	2.7764	-5.5270
3.1747	2.3380	1.2131	1.6632
3.0525	0.0001	0.0186	0.0638
3.1266	0.2470	4.8343	-8.7658
3.1248	0.4949	10.1977	-1.1834

x	y	v_x	v_y
3.1222	0.7461	0.9966	11.6779
3.1233	0.9962	3.3900	8.7395
3.1327	1.2380	-2.5944	-12.7474
3.1300	1.4913	4.5533	-7.9261
3.1402	1.7673	3.8436	4.9984
3.1427	2.0227	1.0952	-1.6182
3.3486	0.1857	-2.4205	1.2963
3.3291	0.3910	0.9393	-2.6814
3.3371	0.6264	6.9648	-7.4181
3.3368	0.8694	-6.4885	-3.3964
3.3464	1.1181	-3.0228	-4.5212
3.3492	1.3579	4.2370	-1.5043
3.3440	1.6219	6.5429	-8.6951
3.3647	1.8763	0.0984	-4.1293
3.2710	0.0001	-0.0161	0.1076
3.3763	2.1674	-4.8366	13.8139
3.5718	2.0153	3.8594	9.8794
3.5208	0.0000	-0.0378	0.0626
3.5837	0.2415	3.0647	9.3964
3.5589	0.4883	-7.1236	-5.0884
3.5488	0.7433	-5.1541	-0.1805
3.5634	0.9844	3.4429	2.9588
3.5660	1.2387	-2.2755	0.2331
3.5541	1.4917	1.6147	-4.9673
3.5615	1.7416	-6.0995	2.2116
3.7432	2.1929	4.0725	5.2158
3.4782	2.3992	1.5604	-5.3767
3.7499	0.0001	-0.0510	0.1765
3.8110	0.1590	1.4695	0.9035
3.7868	0.3921	-5.1621	-0.2439
3.7700	0.6272	0.6524	2.3052
3.7793	0.8703	-2.5478	-9.5304
3.7798	1.1140	-4.2115	10.8396
3.7683	1.3590	3.0682	5.4575
3.7820	1.6267	1.8173	-1.0107
3.7778	1.8804	3.9462	-7.6287
3.9999	0.0000	-0.1073	-0.0488
3.9999	0.2501	0.7621	-0.1870
4.0000	0.4992	-0.0568	0.0233
3.9999	0.7473	-0.0419	0.7002
4.0000	0.9972	0.4418	0.1390
3.9997	1.2473	-0.1036	-0.4293
3.9999	1.4973	-0.0454	-0.3441
4.0000	1.7474	-0.0481	-0.0378
3.9999	1.9973	-0.0477	-0.9605
3.9994	2.2481	-0.0598	-0.6811

position coordinates (2.3) revealed that the particle had crossed JK , then the particle was reset to the nearest point of JK with $\vec{0}$ velocity.

In the remainder of this paper, the particle positions and velocities of Tables 1 and 2 are to be considered the initial data from which wave motions will be generated.

4. Geometry A Results

Since geometry A is physically simplistic, we will couple it with a comparable simplistic approach to wave generation. The primary value of the model is its exceptional economy. Unfortunately, it yields no insight into the mechanism by which earthquakes generate waves. Consider, then, as shown in Figure 4.1, the movement of a rectangular section of earth, with vertices M, N, P and E, into the ocean area. For illustrative purposes, let the coordinates of M and N be (1.1, 0) and (1.1, 0.22), respectively. Next, for each particle P_i with $x_{i,0} \geq 1.1$, reset the particle to $(x_{i,0}, y_{i,0} + 0.22)$, thus displacing vertically all the water directly above the quake. Also, it will be assumed that the shock of the quake has increased various particle velocities, which will be implemented as follows. Let L be the line whose equation is

$$(4.1) \qquad y = 2.42 - 2x$$

and whose graph is shown in Figure 4.1. Then, each particle P_i to the right of L will have its initial velocity incremented, and, for illustrative purposes, we will set up a parabolic profile by using, for a new initial velocity:

$$(4.2) \qquad (v_{i,0,x} - \sqrt{10-x_{i,0}},\ v_{i,0,y} + 5\sqrt{10+x_{i,0}})$$

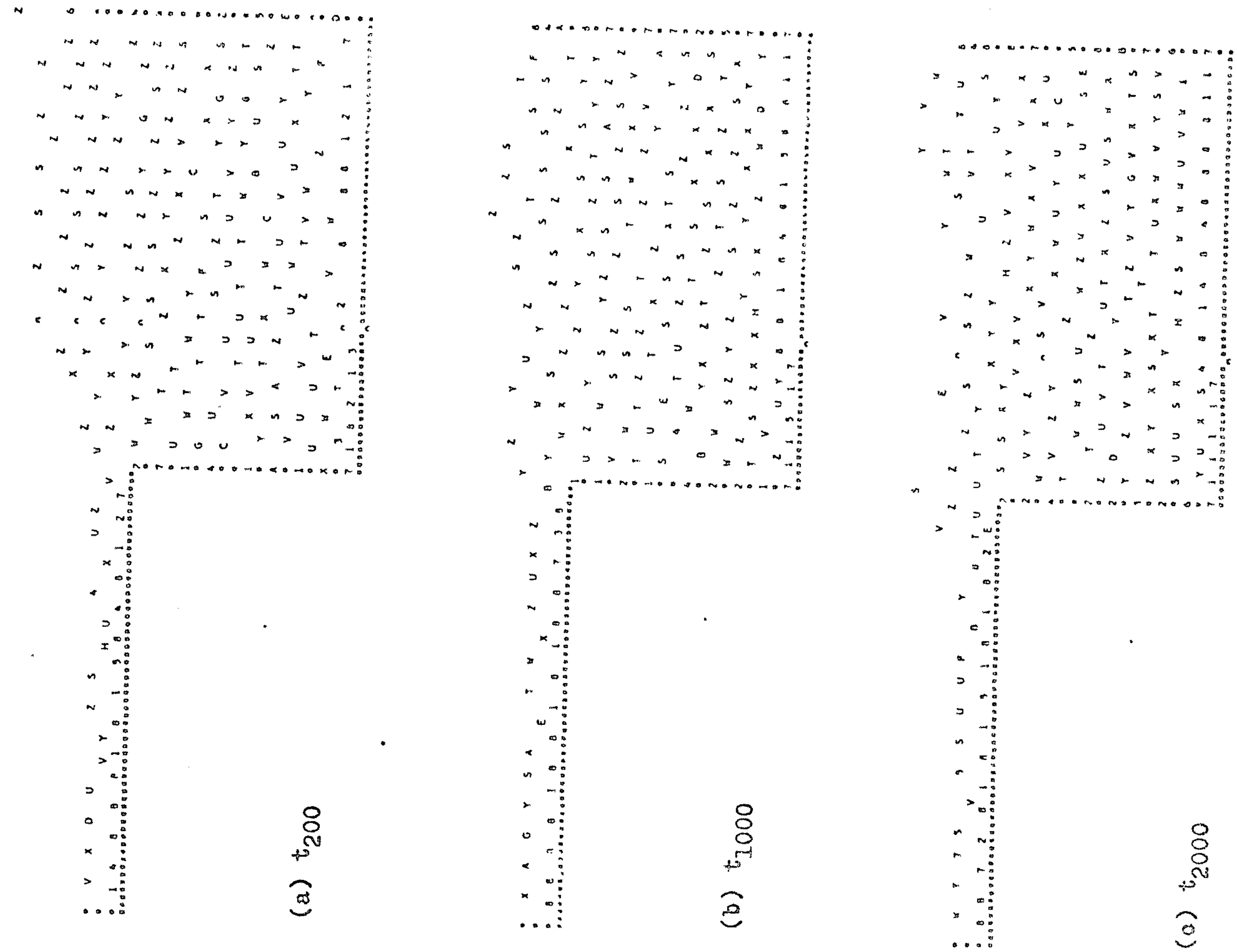
The particles are now allowed to interact in accordance with (2.1)-(2.8), but with the new wall profile being that of Figure 4.1.

The resulting flow is shown in Figure 4.2(a)-(j). Figure (a) shows the ocean deformation due to the quake at the early time t_{200}. Figures (b)-(j) display the wave motions that result after the ocean particles shown in (a) begin to fall. In (c)-(i) is shown the entering and breaking of the first large wave into the bay. In (j) is shown the emergence at t_{4000} of a second, smaller wave into the bay and various dissipating waves in the ocean.

For convenience, the particles have been plotted automatically in Figure 4.2 and in such a fashion that particle velocities can be estimated from the code shown in Figure 4.3, which is explained as follows. Find the particle's number or letter in Figure 4.3. Then the particle's velocity vector is approximately in the direction of the vector from the origin of Figure 4.3 to the given number or letter. All numbers within the inner circle have associated speeds less than unity. All letters in the annular region between the two circles have associated speeds between unity and 5.0. All letters outside the outer circle have associated speeds greater than 5.0. Thus, for example, a particle with designation H is moving upward and to the left with a speed between 1.0 and 5.0. (Unfortunately, the initial automatic plotting routine was not considered to be sufficiently accurate and the plotter itself would plot only zero in one of its columns. The first problem was rectified by writing a better program which was used for all the examples which follow. The second problem, however, was a hardware problem which we could not eliminate.)

With regard to other calculations using the method of this section,

FIGURE 4.2



(a) t_{200}

(b) t_{1000}

(c) t_{2000}

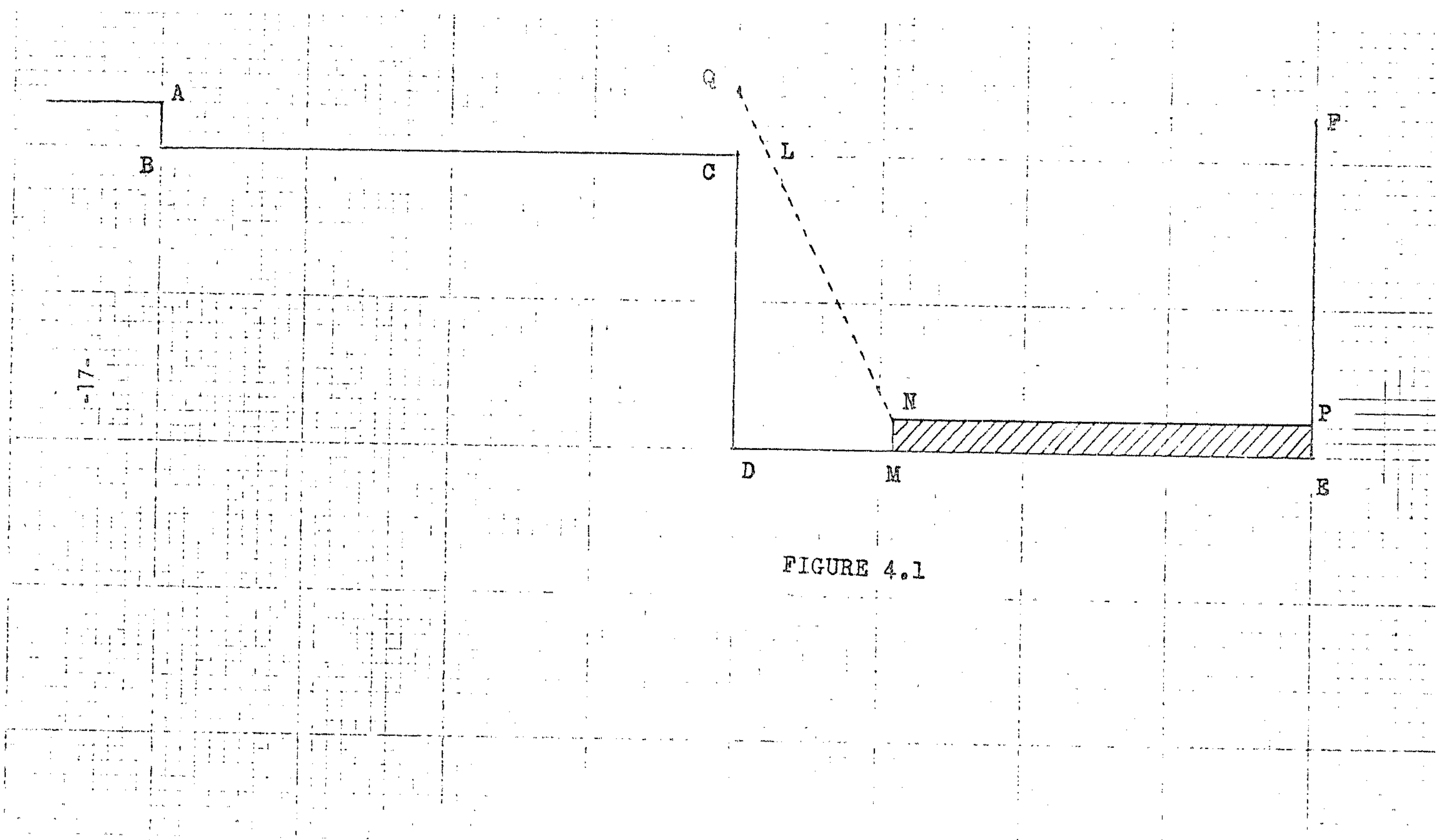
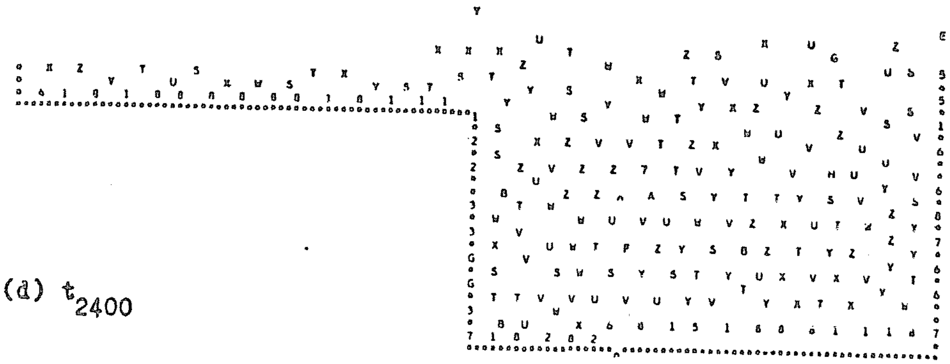
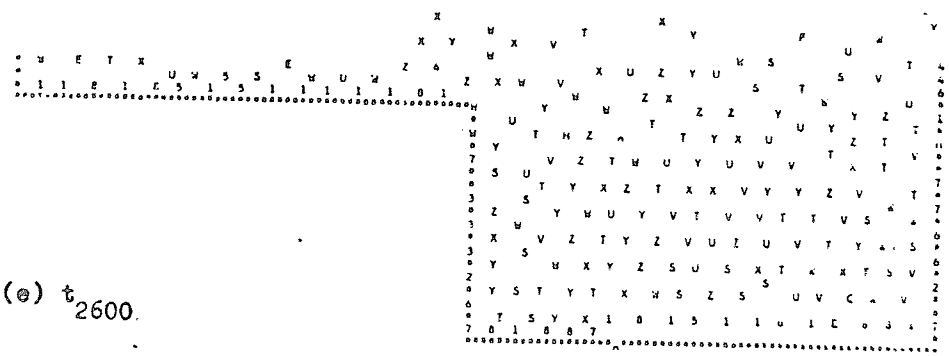


FIGURE 4.1

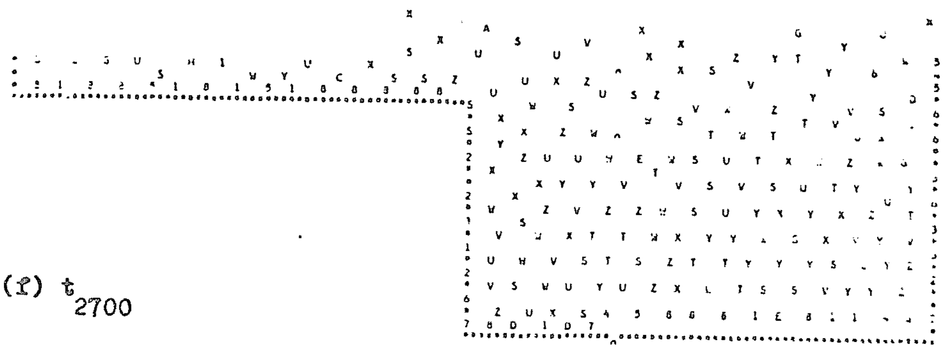
Figure 4.2 (continued)



(d) t_{2400}

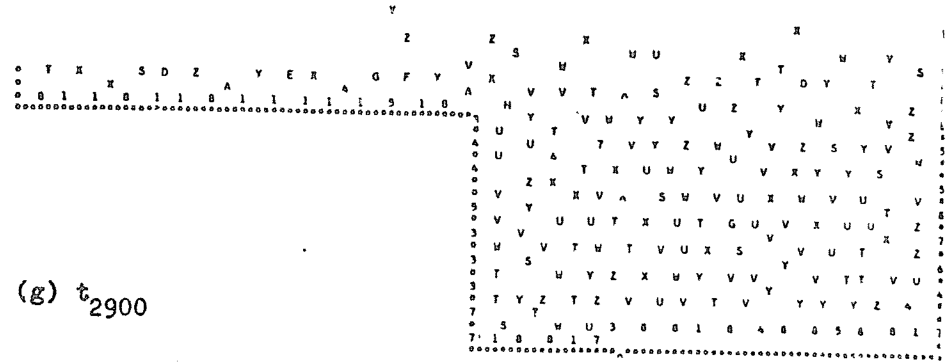


(e) t_{2600}

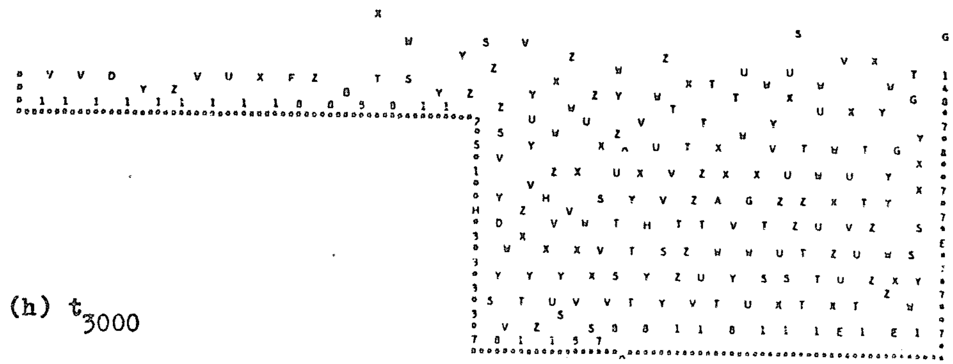


(f) t_{2700}

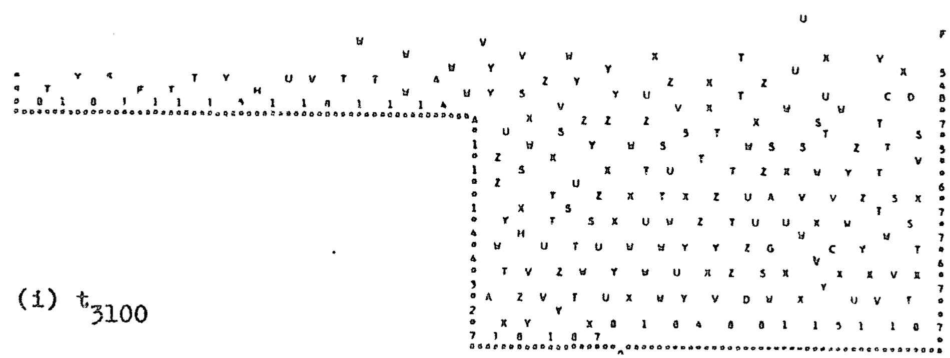
Figure 4.2 (continued)



(g) t_{2900}



(h) t_{3000}



(i) t_{3100}

it should be noted that increasing y coordinates without also increasing particle speeds did not generate waves into the bay, while formulas other than (4.2) but of the general form

$$\vec{v}_{i,0} = (v_{i,0,x} + \alpha\sqrt{\beta \pm x_i},\ v_{i,0,y} + \gamma\sqrt{\delta \pm x_i})$$

could be used to yield reasonable wave motions.

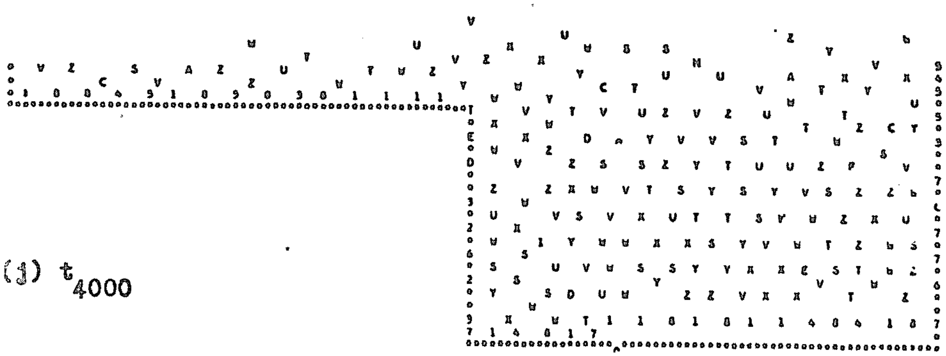
5. Geometry B Results

Though examples using the technique of Section 4 were also generated for geometry B, we will concentrate in this section on a more realistic approach, since geometry B more realistically represents a continental shelf formation than does geometry A. For this purpose, consider a movement of earth into the ocean sector as shown in Figure 5.1. Let the coordinates of M, N and P be $(a,0)$, (a,α) and $(4.0, \beta)$, respectively. Water particles which were originally in the region EMNP will simply be relocated vertically onto the segment NP, thus creating a region of high fluid compression, and, hence, increased potential energy, along this interface. The particles are then allowed to interact in accordance with (2.1)-(2.8), but with the new wall profile being that of Figure 5.1. Collision with earthquake wall NP will be treated simply by merely resetting the particle vertically onto NP, resetting its velocity components to one-tenth their incident values, and changing the sign of the y component of velocity.

In order to explore the mechanism of wave generation and the resulting waves themselves, let us consider an accentuated example in which the earthquake is relatively large. For this purpose, set $a = 2.1$, $\alpha = 0.08$ and $\beta = 0.15$. In Figure 5.2(a), one sees that at t_{25} the potential energy at the interface is transforming rapidly into kinetic energy. Though the circled particles all, according to the code of Figure 4.3, are moving upward with speeds greater than 5, a detailed printout reveals that, in fact, the average speed of these particles at this early time is closer to 50 units per second. Figures 5.2(b)-(d). Then show the

Figure 4.2 (continued)

(j) t_{4000}



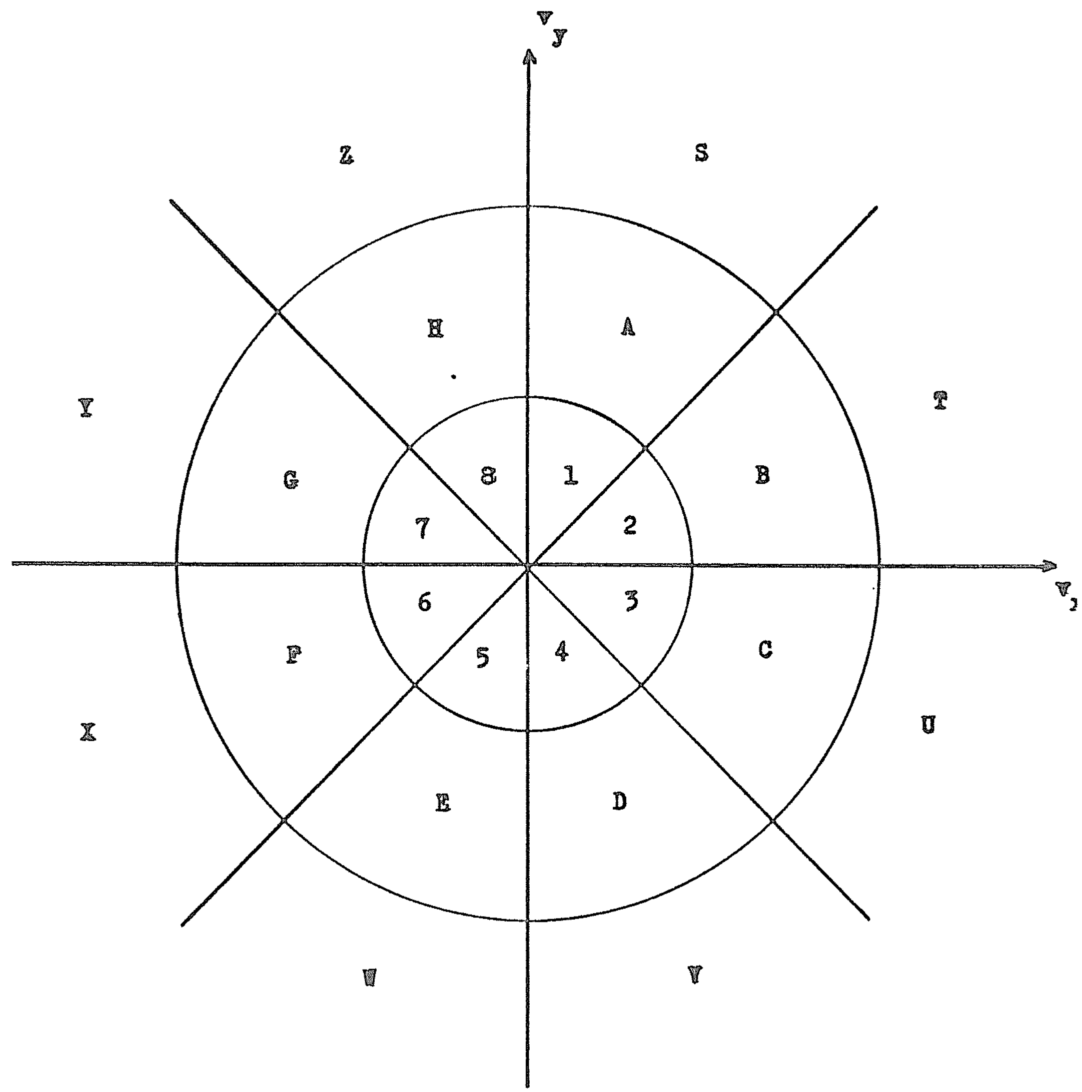
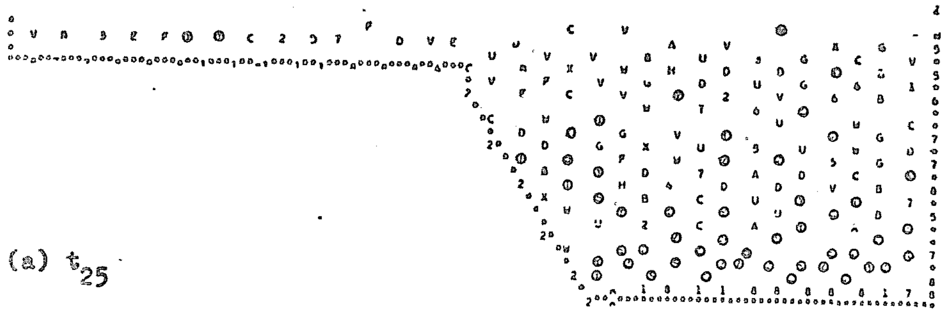
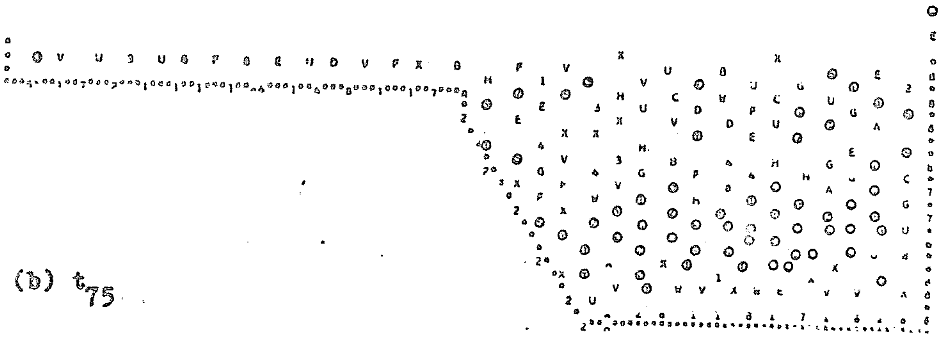


FIGURE 4.3

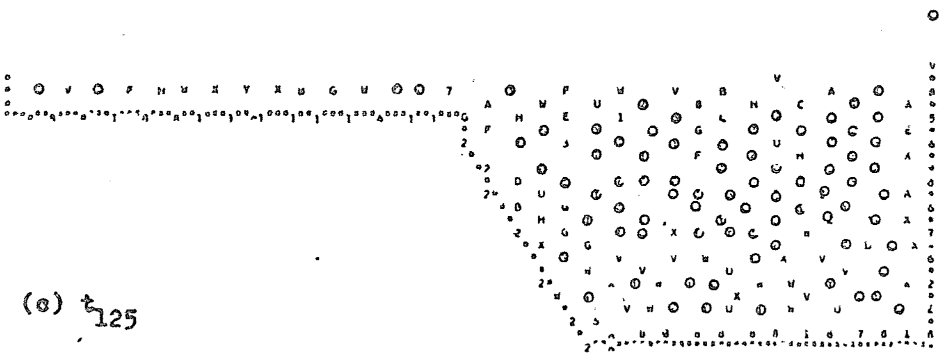
FIGURE 5.2



(a) t_{25}

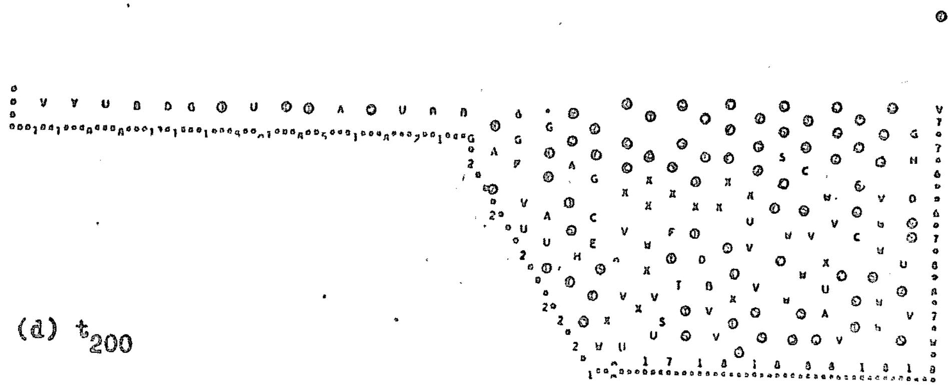


(b) t_{75}

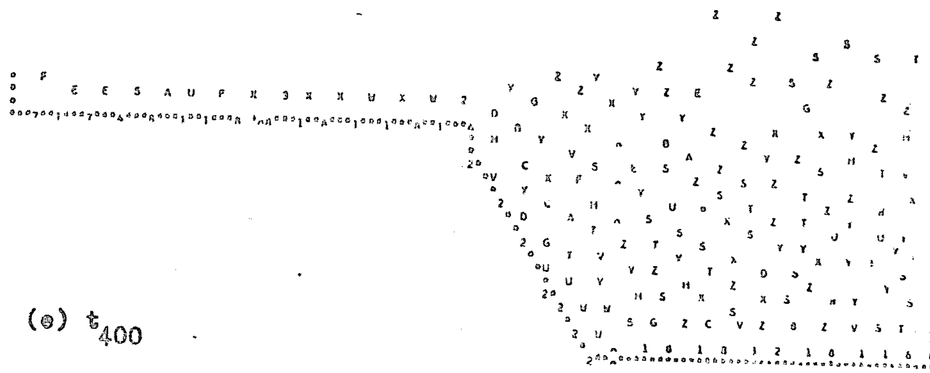


(c) t_{125}

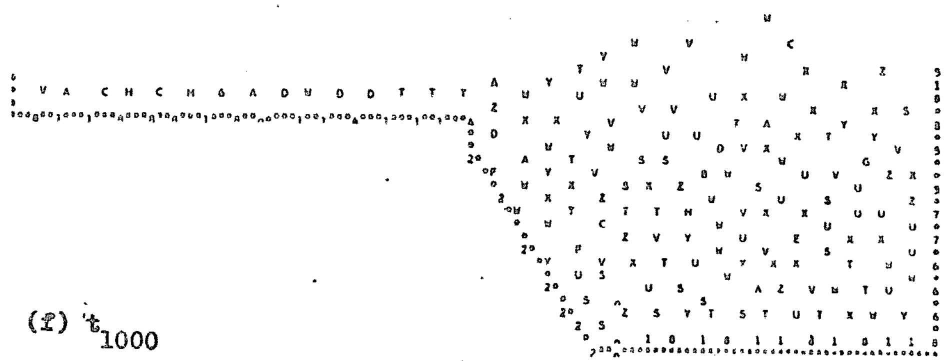
FIGURE 5.2 (continued)



(d) t_{200}



(e) t_{400}



(f) t_{1000}

transfer of this kinetic energy, by a compression wave, up to the surface of the ocean at time t_{200} . The circled particles, again, are those moving upward with speeds greater than 5. However, in Figure 5.2(d), so many particles are now included in the energy transfer that the average speed of those particles at, or just below, the surface is only about 7.5. Figures 5.2(e)-(n) then show the resulting wave patterns. In particular, (g)-(j) show the first wave to break into the bay, (k)-(m) show new waves entering the bay and waves within the bay, itself, while (n) at t_{4900} shows the flow of ocean water over the bay water as the ocean itself begins to reach a more stable configuration.

Completely analogous, but somewhat less dramatic, examples were run with $\alpha = \beta = .08$ and $\alpha = \beta = .12$. In all cases, the generation of surface waves by an initial compression wave was clearly visible. This mechanism is of fundamental interest because once one makes the assumption of incompressibility for water, as is usual in continuous models, this mechanism is no longer discernable.

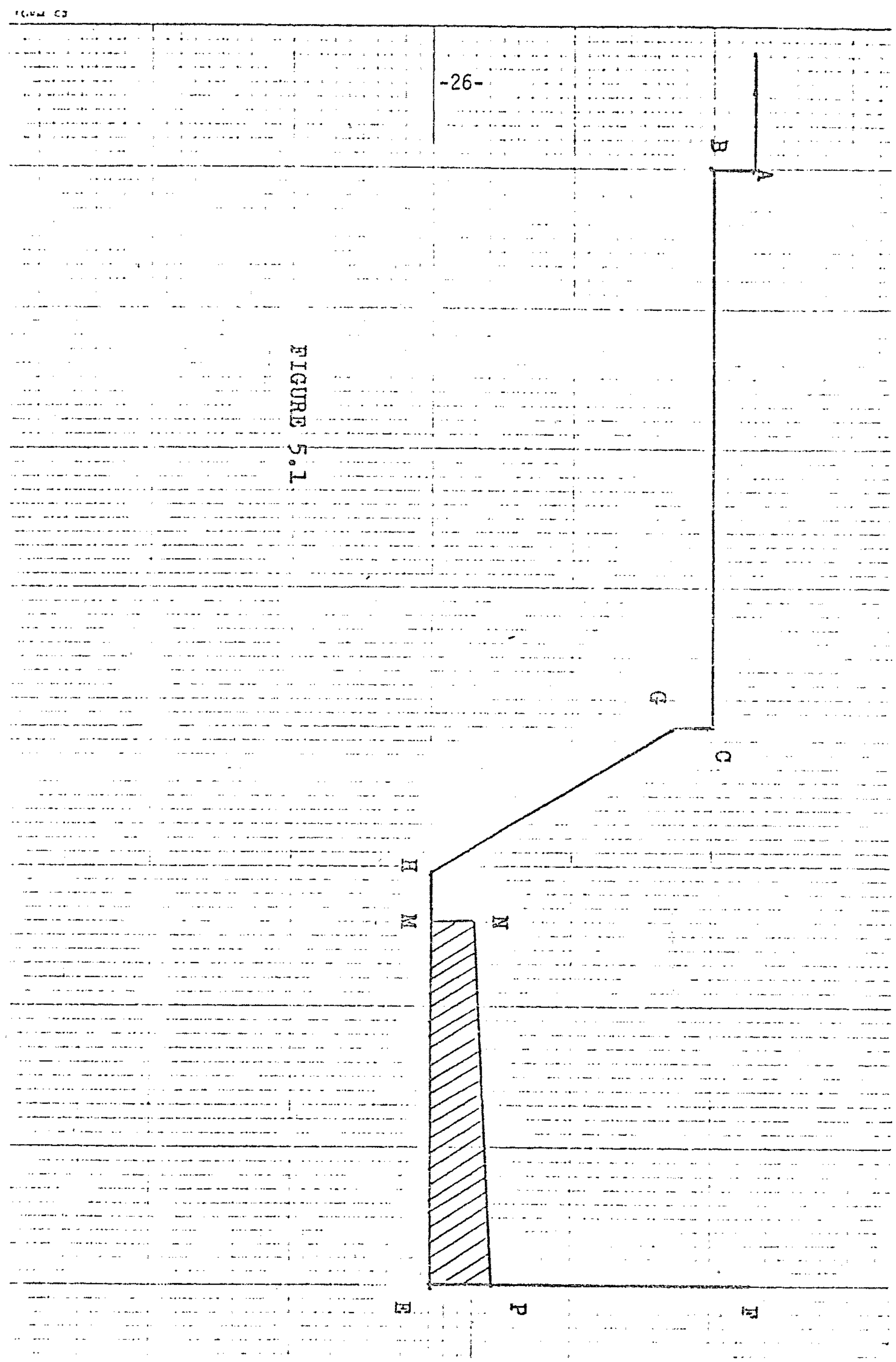
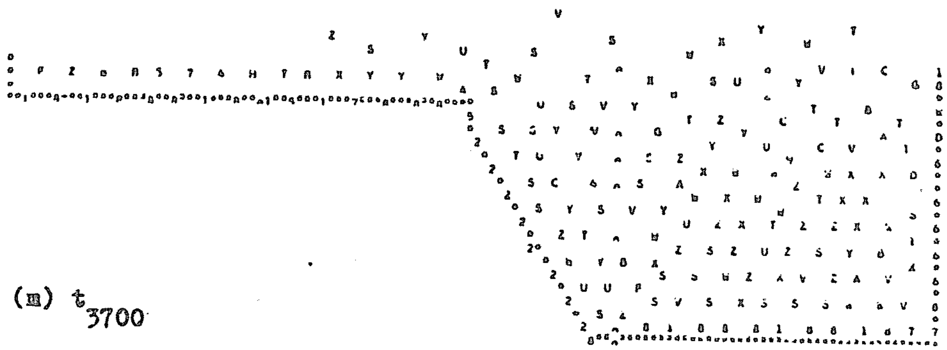
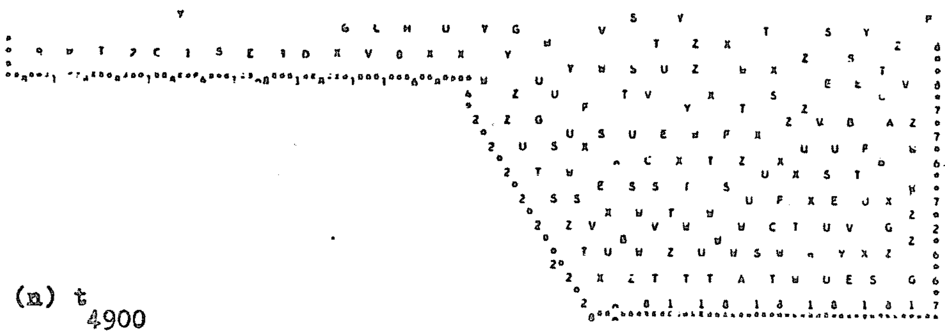


FIGURE 5.1

FIGURE 5.2 (continued)



(m) t_{3700}



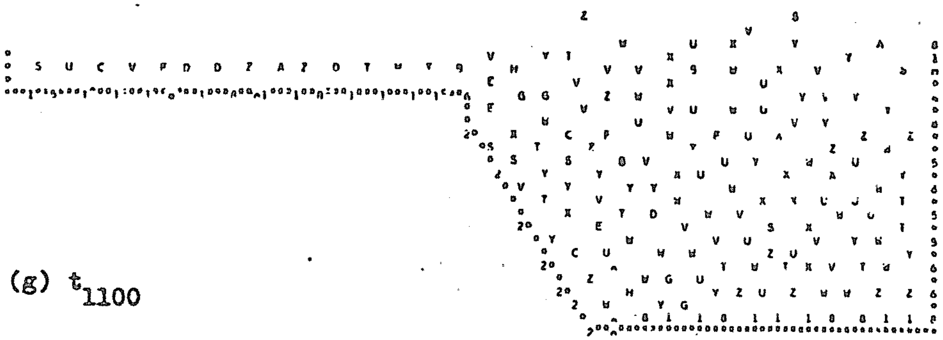
(n) t_{4900}

6. Geometry C Results

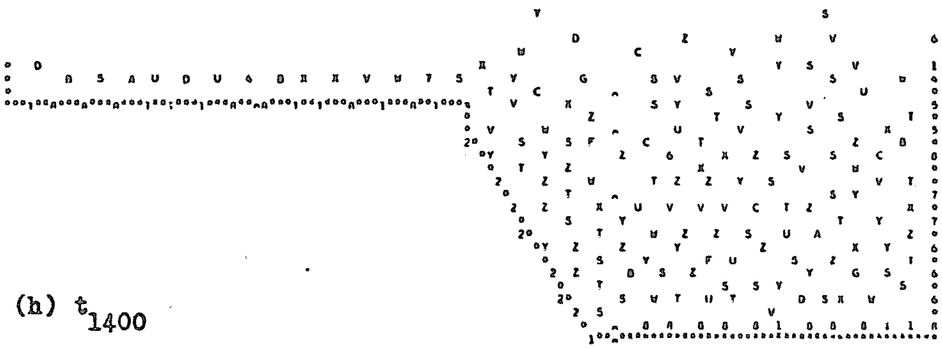
In many cases where earthquake-born ocean waves have had disastrous effects, like in Japan, the continental shelf is most realistically approximated by geometry C. So, in this section we will be even more realistic than in Section 5 by generating the earthquake in a sequence of steps. Specifically, the quake will be generated from $x = 2.5$ to $x = 4.0$ as a rectangular earth mass which rises to $y = .25$ through the values $y = .16$ and $y = .22$. At each intermediate height, the interface is held for five time steps before it is incremented further. The results are shown in Figure 6.1(a)-(h), where large ocean waves and wave flow into the bay are evident.

Besides similar computational results which were obtained for other earthquake heights, particles which collided with the sloped wall were also allowed to reflect with nonzero velocities. In these cases a back flow developed which damped the waves which were near and moving toward the bay. This suggests that the implantation of reflecting surfaces into the sloped wall might act as a deterrent to the development of waves which are so large that they can do excessive damage along the coast.

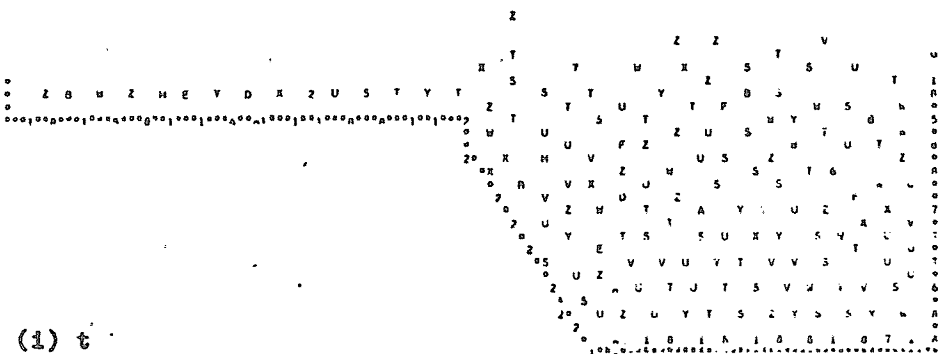
FIGURE 5.2 (continued



(g) t_{1100}

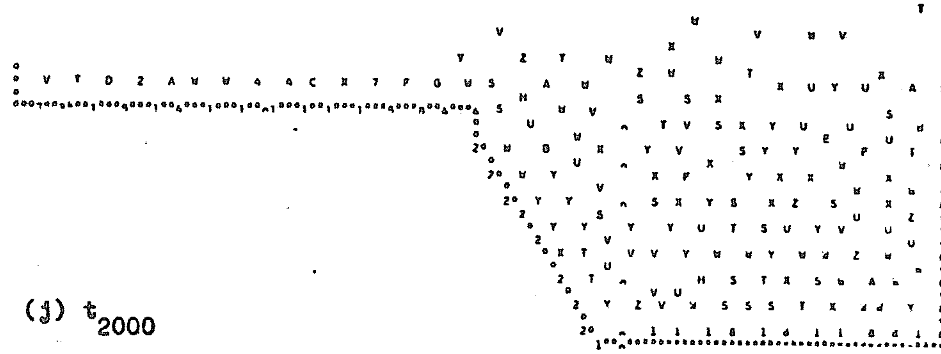


(h) t_{1400}

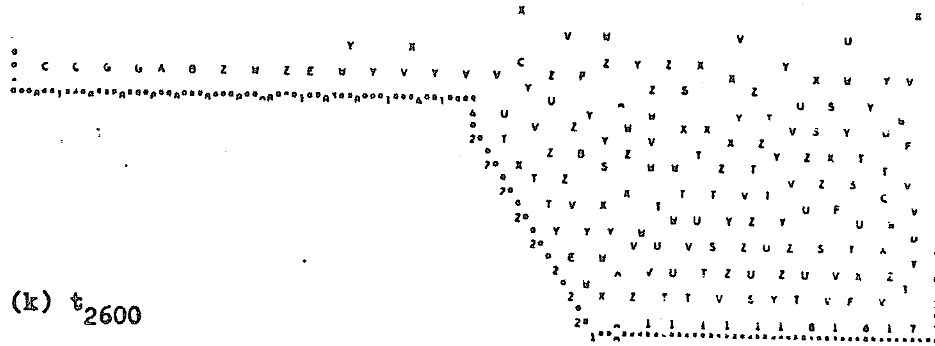


(i) t_{1600}

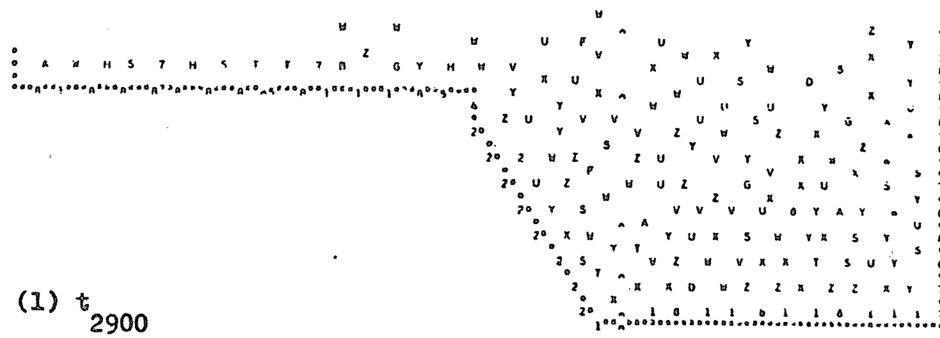
FIGURE 5.2 (continued)



(j) t_{2000}

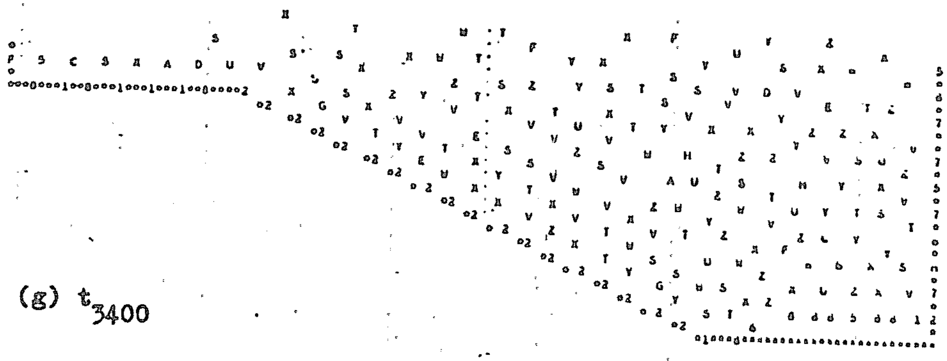


(k) t_{2600}

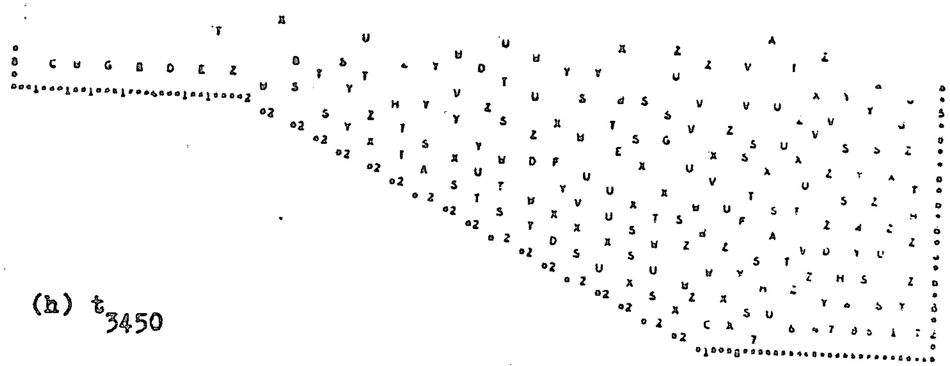


(l) t_{2900}

FIGURE 6.1 (continued)



(g) t_{3400}



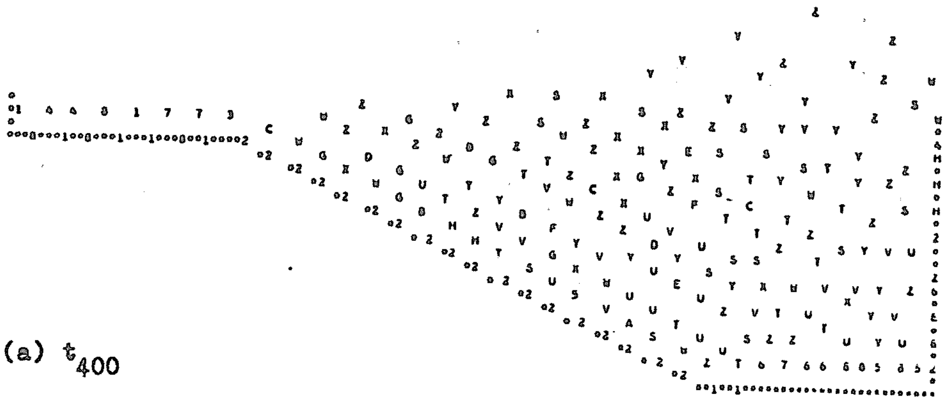
(h) t_{3450}

7. Conclusions and Further Problems

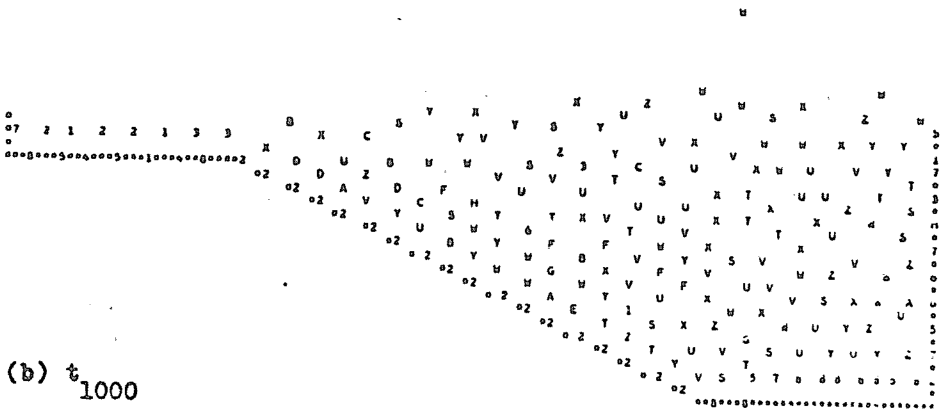
From the model in this paper, we conclude that the mechanism for earthquake generated ocean waves is a complex sequence of compression waves on the molecular level. The nature of the resulting ocean waves depends directly upon the amount of energy imparted to the ocean by the quake and by the particular parameters of the ocean topography under consideration.

With regard to future problems, we intend to concentrate on the study of tsunamis. Since many of the parameters are known for the Tokyo Area [6], we will begin with the topography of this region, attempt to generate an ocean with many more particles than those in Sections 4-6, and generate the quakes at the proportionate distance from the bay. This will require also having an ocean sector to the right of the quake, unlike the-examples of Sections 4-6. We will also probe the possibility of elastic behavior in the bay floor in order to more fully understand the flow out of the bay which precedes a tsunami. Downward earthquake motions will be explored also. Finally, if at all possible, we intend to generate a three-dimensional model.

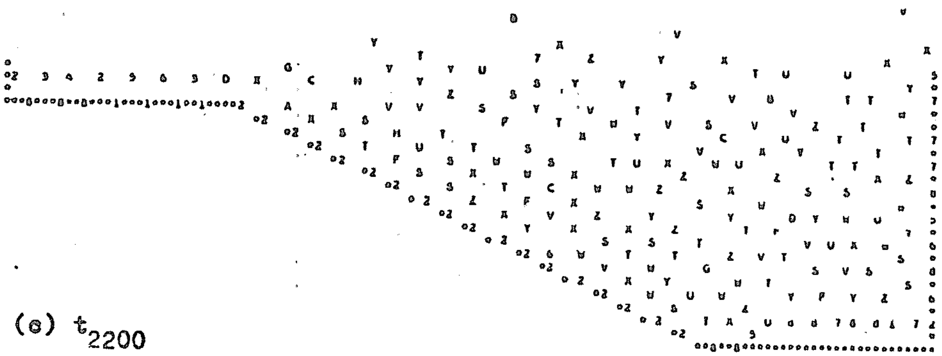
FIGURE 6.1



(a) t_{400}

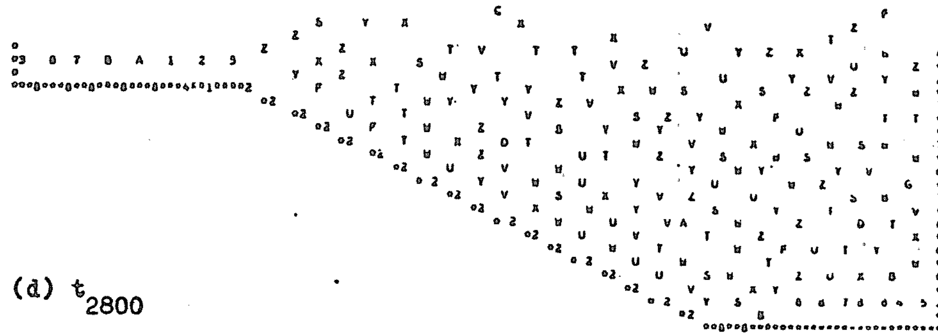


(b) t_{1000}

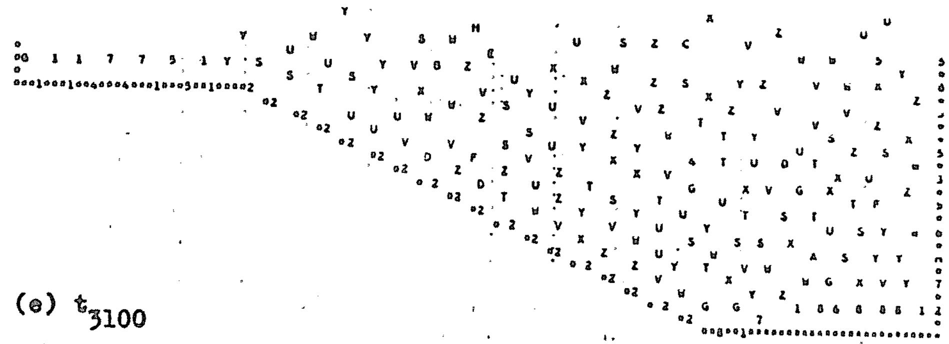


(c) t_{2200}

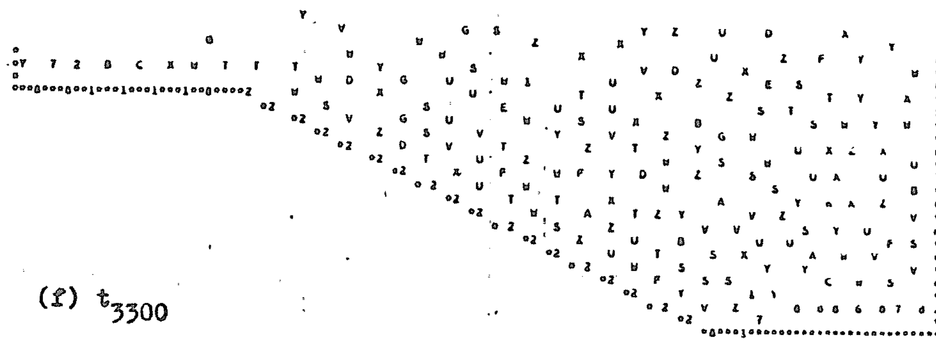
FIGURE 6.1 (continued)



(d) t_{2800}



(e) t_{3100}



(f) t_{3300}

APPENDIX

```
C TSUNAMI - FLAT BAY, DEEP OCEAN - LEAP FROG FORMULAS - INPUT
C IS AT HALF STEPS SO SPECIAL STARTING FORMULAS ARE NOT NEEDED - TIME STEP
C IS .0001 - H=G=1, V=100 - EXPONENT OF REPULSION IS 3 AND APPEARS AS 4
C BECAUSE OF COMPONENT FACTORS X/R, Y/R - EXPONENT OF ATTRACTION IS 1.
C DAMPING FACTOR IS 0.1 AND APPEARS IN VELOCITY REFLECTIONS OF WALCOL
C ONLY NEAR PARTICLES (R.LT.0.25) ARE COUNTED IN FORCE CALCULATIONS
C  IN THIS PART OF THE PROGRAM THE OCEAN HAS STABILIZED AND THE EARTHQUAKE
C  HAS BEEN INITIATED FROM X=1.1 TO X=4.  EACH PARTICLE HAS BEEN RAISED
C .22 UNITS AND ITS VELOCITY, IN ORDER TO MAKE A LONG AND FLAT WAVE, HAS
C BEEN CHANGED.
      PARAMETER N=221
      NM1=N-1
      K=1
      KPRINT=1
C DIMENSION STATEMENT
      DIMENSION PMASS(N),XO(N),YO(N),VXO(N),VYO(N),X(N,2),Y(N,2),
     1VX(N,2),VY(N,2),ACX(N),ACY(N)
C  VARIABLES XO, YO, ETC., HAVE BEEN PUNCHED OH, NOT ZERO
C INITIALIZATION SET TO ZERO AUTOMATICALLY ON 1110
C INITIAL DATA INPUT
      READ 10, (PMASS(I),XO(I),YO(I),VXO(I),VYO(I),I=1,N)
10     FORMAT (5F10.4)
      DO 11 I=1,N
      IF (YO(I).LT.4.42-2.*XO(I)) GO TO 11
      VYO(I)=VYO(I)+5.*SQRT(XO(I)+10.)
      VXO(I)=VXO(I)-1.*SQRT(5.-XO(I))
11      CONTINUE
      DO 12 I=1,N
      IF (XO(I).LT.1.1) GO TO 12
      YO(I)=YO(I)+.22
12      CONTINUE
C PRINT INITIAL DATA
      DO 20 I=1,N
      PRINT 15,I,PMASS(I),XO(I),YO(I),VXO(I),VYO(I)
15     FORMAT (5X,I5,5F11.5)
20      CONTINUE
C UPDATE.  X(I,1) IS X COORDINATE AT PREVIOUS TIME STEP. X(I,2) IS X
C  COORDINATE AT PRESENT TIME STEP.  SIMILARLY FOR OTHER VARIABLES.
      DO 30 I=1,N
      X(I,1)=XO(I)
      Y(I,1)=YO(I)
      VX(I,1)=VXO(I)
      VY(I,1)=VYO(I)
30      CONTINUE
       GO TO 71
65      DO 70 I=1,N
      X(I,1)=X(I,2)
      Y(I,1)=Y(I,2)
      VX(I,1)=VX(I,2)
      VY(I,1)=VY(I,2)
70      CONTINUE
C CALCULATION OF ACCELERATIONS IS DONE THROUGH STEP 770
      DO 701 I=1,N
      ACX(I)=0.
      ACY(I)=0.
701     CONTINUE
```



```
71      DO 77 I=1,NM1
      IP1=I+1
      DO 76 J=IP1,N
      R=SQRT((X(I,1)-X(J,1))**2+(Y(I,1)-Y(J,1))**2)
      IF (R.GT.0.25) GO TO 73
      FX=PMASS(I)*PMASS(J)*(X(I,1)-X(J,1))*(1.-R*R)/R**4
      FY=PMASS(I)*PMASS(J)*(Y(I,1)-Y(J,1))*(1.-R*R)/R**4
      GO TO 75
73      FX=0.
      FY=0.
C ACCUMULATION OF FORCES ON PARTICLE I DUE TO ALL OTHER PARTICLES IS DONE
C IN NEXT FOUR FORMULAS
75      ACX(I)=ACX(I)+FX
      ACX(J)=ACX(J)-FX
      ACY(I)=ACY(I)+FY
      ACY(J)=ACY(J)-FY
76      CONTINUE
77      CONTINUE
C NOTE THAT WE HAVE JUST ACCUMULATED FORCES, NOT ACCELERATIONS - THE ABOVE
C NOTATION, THOUGH MISLEADING, ENABLES US TO SAVE MEMORY LOCATIONS.  WE
C NOW CALCULATE THE ACCELERATIONS
      DO 770 I=1,N
      ACX(I)=ACX(I)/PMASS(I)
      ACY(I)=(ACY(I)/PMASS(I))-980.
770      CONTINUE
79      DO 80 I=1,N
      VX(I,2)=VX(I,1)+.0001*ACX(I)
      VY(I,2)=VY(I,1)+.0001*ACY(I)
      X(I,2)=X(I,1)+.0001*VX(I,2)
      Y(I,2)=Y(I,1)+.0001*VY(I,2)
80      CONTINUE
      CALL WALCOL
      K=K+1
C PRINT ONLY EVERY KPRINT STEPS
      IF (MOD(K,KPRINT).GT.0) GO TO 82
      IF (MOD(K,KPRINT).LT.0) GO TO 82
      DO 810 I=1,N
      PRINT 81,K,I,X(I,2),Y(I,2),VX(I,2),VY(I,2)
81      FORMAT (5X,2I5,4F20.10)
810      CONTINUE
C CALCULATE BAY AND TOTAL KE
C IT IS NECESSARY THAT CARDS 1-29 BE BAY DATA CARDS
      ENERGY=0.
      DO 8100 I=1,29
      ENERGY = ENERGY+50.*(VX(I,2)**2+VY(I,2)**2)
8100    CONTINUE
      PRINT 8101, ENERGY
8101    FORMAT (5X,F20.10)
      DO 8105 I=30,N
      ENERGY = ENERGY+50.*(VX(I,2)**2+VY(I,2)**2)
8105    CONTINUE
      PRINT 8106, ENERGY
8106    FORMAT (5X,F20.10)
C TERMINATION AFTER A FIXED NUMBER OF STEPS
82      IF (K.LT.5) GO TO 65
C PUNCH OUTPUT FOR RESTART
```

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