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STABILITY ANALYSIS IN DISCRETE MECHANICS

by

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STABILITY ANALYSIS IN DISCRETE MECHANICS*

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1. Introduction

Greenspan [2, 3] has developed a discrete version of Newtonian mechanics which makes it possible for problems in mechanics to be formulated in a form suitable for solution on a digital computer.

In applying discrete mechanics, it is necessary to use a time-step Δt which is small enough to ensure stability (Greenspan [2, p. 22; 3, p. 19]). In this report we analyze in detail the stability properties of one of the problems treated by Greenspan. For this problem (which we describe in section 2), Greenspan found experimentally that the discrete mechanics model was stable if

$$\Delta t \leq 2\alpha, \tag{1.1}$$

where α is a positive constant associated with the problem. Our main result, which is proved in section 5, is that the discrete mechanics model is stable if

$$\Delta t < \min(2\alpha, 2/\alpha). \tag{1.2}$$

Since in Greenspan's experiments α was always less than or equal to 1, our result confirms Greenspan's experimental observations.

We also prove certain other results: in section 3 we show that the discrete mechanics model is convergent; and in section 4 we show

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that the linearized discrete mechanics model is stable if and only if

$$\left.\begin{array}{l} \Delta t \leq \min(2\alpha, 2/\alpha), \quad \text{for} \quad \alpha \neq 1 \\ \Delta t < 2, \quad \text{for} \quad \alpha = 1. \end{array}\right\} \qquad (1.3)$$

We have emphasized the close relationship between the "continuous analysis" of section 2 and the "discrete analysis" of section 5, in the hope that this will clarify the proofs and perhaps make it possible to adapt the proofs for other problems.

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2. Newtonian Mechanics

The problem to be considered (Greenspan [2, p. 18]) is the damped motion of a particle of unit mass which is constrained to move with its center on the x-axis. We denote the position and velocity of the particle by x and v, respectively. The particle is subjected to a viscous damping force αv, where α is a positive constant, and a restraining force $\sin x$. At time $t = 0$ the particle is at x_0 and has velocity v_0.

In Newtonian mechanics the problem can be formulated as follows:

$$\frac{dx}{dt} = v, \qquad (2.1)$$

$$\frac{dv}{dt} = -(\alpha v + \sin x), \qquad (2.2)$$

$$x(0) = x_0, \qquad v(0) = v_0. \qquad (2.3)$$

Since the right hand sides of (2.1) and (2.2) are Lipschitz continuous, the solution $x(t)$, $v(t)$, exists for all $t \geq 0$. (Coddington and Levinson [1, p. 20]).

Theorem 2.1.

If $x(t)$, $v(t)$, is the solution of the initial value problem {(2.1), (2.2), (2.3)} then, as $t \to \infty$,

$$v(t) \to 0, \tag{2.4}$$

and

$$x(t) \to k\pi, \tag{2.5}$$

where k is an integer. In particular, if $v_0 = 0$ and $|x_0| < \pi$, then $k = 0$.

Proof:

Multiplying (2.2) by v, and using (2.1), we find that

$$\frac{dE}{dt} = -\alpha v^2, \tag{2.6}$$

where

$$E(t) = v^2/2 - \cos x. \tag{2.7}$$

Clearly, $E(t)$ is the energy of the particle, and consists of the sum of the kinetic energy, $v^2/2$, and the potential energy, $-\cos x$.

Integrating (2.6),

$$E(t) + F(t) = E(0), \tag{2.8}$$

where

$$F(t) = \alpha \int_0^t [v(s)]^2 \, ds. \tag{2.9}$$

Several important conclusions follow from (2.8). Since $F(t) \geq 0$,

$$E(t) = v^2/2 - \cos x \leq E(0).$$

Hence,

$$|v(t)| \leq V = [2(1 + E(0))]^{1/2}. \tag{2.10}$$

Since $E(t) \geq -1$,

$$F(t) \leq 1 + E(0).$$

Remembering that $F(t)$ is a monotone increasing function of t, we can conclude that $F(t)$ converges monotonely to a finite constant L,

$$F(t) \uparrow L. \tag{2.11}$$

Finally, combining (2.1) and (2.10), we see that

$$|x(t') - x(t'')| \leq V|t' - t''|, \tag{2.12}$$

for all t', t''.

We now prove (2.4), namely that $v(t) \to 0$. Suppose that this is not the case. Then there is a constant β,

$$0 < \beta < V/\alpha, \tag{2.13}$$

such that for any $t_1 \geq 0$, there exists a $t_2 > t_1$ such that

$$[v(t_2)]^2 \geq \beta. \tag{2.14}$$

We prove (2.4) by showing that (2.14) leads to a contradiction.

From (2.11) it follows that is a t_1 such that

$$L - \alpha\beta^2/(32V) \leq F(t) \leq L, \quad \text{for} \quad t \geq t_1. \tag{2.15}$$

Let $t_2 > t_1$ be such that (2.14) holds. Then, using (2.7), (2.8), (2.12), (2.13), (2.14), and (2.15),

$$\begin{aligned}
[v(t)]^2/2 &= \cos[x(t)] + E(t), \\
&= \cos[x(t)] + E(0) - F(t), \\
&= \{\cos[x(t)] - \cos[x(t_2)]\} + \{F(t_2) - F(t)\} + [v(t_2)]^2/2, \\
&\geq -|x(t) - x(t_2)| - \alpha\beta^2/(32V) + \beta/2, \\
&\geq -V|t - t_2| - \beta/32 + \beta/2.
\end{aligned} \tag{2.16}$$

Setting

$$t_3 = t_2 + \beta/(8V), \tag{2.17}$$

we see from (2.16) that

$$[v(t)]^2 \geq \beta/2, \quad \text{for} \quad t_2 \leq t \leq t_3. \tag{2.18}$$

Hence, noting (2.9) and (2.15),

$$\begin{aligned}
F(t_3) &= F(t_2) + \alpha \int_{t_2}^{t_3} [v(s)]^2 \, ds, \\
&\geq L - \alpha\beta^2/(32V) + \alpha\beta^2/(16V), \\
&> L,
\end{aligned}$$

contradicting (2.15). We have thus proved (2.4).

To prove (2.5) we note that since $F(t) \to L$ and $v(t) \to 0$, it follows from (2.7) and (2.8) that

$$\cos[x(t)] \quad \to \quad -E(0) + L. \tag{2.19}$$

Together, (2.19) and (2.12) imply that

$$x(t) \to \bar{x}, \tag{2.20}$$

for some constant $\bar{x}$. Thus from (2.2),

$$\frac{dv}{dt} \to -\sin \bar{x}. \tag{2.21}$$

Since $v \to 0$, we see that $\sin \bar{x} = 0$, so that we have proved (2.5).

Finally, we consider the special case when $v_0 = 0$ and $|x_0| < \pi$. Then, from (2.8) and (2.9), $E(t) \leq E(0)$. Hence, from (2.7), $\cos[x(t)] \geq -E_0 + [v(t)]^2/2 = \cos(x_0) + [v(t)]^2/2 > -1$.

Since $x(t)$ is a continuous function, and $|x(0)| = |x_0| \leq \pi$, it follows that $|x(t)| < \pi$ for all t. Hence, $k = 0$.

3. Discrete mechanics: convergence

For the problem considered, Greenspan [2, p. 20] used the computational scheme,

$$\begin{aligned} x_n = {} & (3 - \alpha\Delta t)x_{n-1} + (-1)^{n-1}(2 - \alpha\Delta t)x_0 \\ & + (2\alpha\Delta t - 4)\sum_{j=2}^{n-1}[(-1)^j x_{n-j}] \\ & + (-1)^{n-1}(1 - \alpha\Delta t/2)v_0\,\Delta t - \frac{(\Delta t)^2}{2}\sin x_{n-1}. \end{aligned} \tag{3.1}$$

Noting equations (7.1), (7.4), and 9.2), of Greenspan [2], we see that the scheme

$$\left.\begin{aligned} x_{n+1} &= x_n + \Delta t(v_n + v_{n+1})/2, \\ v_{n+1} &= v_n - \Delta t(\alpha v_n + \sin x_n), \end{aligned}\right\} \tag{3.2}$$

is equivalent to (3.1), and we will use (3.2) in our analysis.

The scheme (3.2) is clearly an approximation to the initial value problem {(2.1), (2.2), (2.3)}. Using the notation of Henrici [4], the increment function Φ (Henrici [4, p. 117]) is given by

$$\Phi(x,v;\Delta t) = \begin{pmatrix} v - \Delta t(\alpha v + \sin x)/2 \\ -(\alpha v + \sin x) \end{pmatrix} .$$

Since $\Phi(x,v;0) = \begin{pmatrix} v \\ -(\alpha v + \sin x) \end{pmatrix}$,

the scheme (3.2) is consistent (Henrici [4, p. 124]) with (2.1), (2.2), and (2.3).

From Henrici [4, p. 124] we obtain

Theorem 3.1.

The scheme (3.2) is convergent. That is,

$$\lim_{\substack{\Delta t \to 0 \\ n\Delta t = t}} \begin{pmatrix} x_n \\ v_n \end{pmatrix} = \begin{pmatrix} x(t) \\ v(t) \end{pmatrix} ,$$

where $x(t)$, $v(t)$, is the solution of the initial value problem {(2.1), (2.2), (2.3)}.

4. Discrete mechanics: stability of the linearized model

In Theorem 2.1 we proved that, in certain cases, $x(t) \to 0$ as $t \to \infty$. When analyzing the discrete mechanics model, it is therefore reasonable to begin by considering the case when x_n is small. In this case, equations (3.2) become, approximately,

$$\left.\begin{aligned} x_{n+1} &= x_n + \Delta t(v_n + v_{n+1})/2, \\ v_{n+1} &= v_n - \Delta t(\alpha v_n + x_n). \end{aligned}\right\} \qquad (4.1)$$

The following theorem gives necessary and sufficient conditions for the stability of the scheme (4.1):

<u>Theorem 4.1.</u>

The solution x_n, v_n, of (4.1) is bounded as $n \to \infty$ for all initial values x_0, v_0, iff

$$\left.\begin{aligned} &\Delta t \leq \min(2\alpha, 2/\alpha), \quad \text{if} \quad \alpha \neq 1, \\ &\Delta t < 2, \quad \text{if} \quad \alpha = 1. \end{aligned}\right\} \qquad (4.2)$$

<u>Proof:</u>

According to the standard theory of linear difference equations, if λ_1 and λ_2 are the zeros of the polynomial

$$\begin{vmatrix} \lambda - 1, & -\Delta t(1+\lambda)/2 \\ \Delta t, & \lambda - 1 + \alpha\Delta t \end{vmatrix} = \lambda^2 + \lambda(-2 + \alpha\Delta t + [\Delta t]^2/2) + (1 - \alpha\Delta t + [\Delta t]^2/2), \qquad (4.3)$$

then the general solution of (4.1) is of the form

$$\begin{pmatrix} x_n \\ v_n \end{pmatrix} = \begin{cases} A\lambda_1^n \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} + B\lambda_2^n \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}, & \text{for} \quad \lambda_1 \neq \lambda_2, \\ A\lambda_1^n \begin{pmatrix} a_1 \\ b_1 \end{pmatrix} + nB\lambda_1^{n-1} \begin{pmatrix} a_2 \\ b_2 \end{pmatrix}, & \text{for} \quad \lambda_1 = \lambda_2, \end{cases} \qquad (4.4)$$

where $\begin{pmatrix} a_1 \\ b_1 \end{pmatrix}$ and $\begin{pmatrix} a_2 \\ b_2 \end{pmatrix}$ are certain linearly independent vectors.

From (4.4) we see that x_n and v_n will be bounded as $n \to \infty$ for all x_0, v_0, iff

$$\left.\begin{array}{ll} \text{(a)} & |\lambda_1| \leq 1, \quad |\lambda_2| \leq 1, \\ \text{(b)} & \text{if } |\lambda_1| = |\lambda_2| = 1, \text{ then } \lambda_1 \neq \lambda_2. \end{array}\right\} \tag{4.5}$$

Using the results of the Appendix, we see that (4.5a) is satisfied iff

$$|1 - \alpha\Delta t + [\Delta t]^2/2| \leq 1, \tag{4.6}$$

and $$|-2 + \alpha\Delta t + [\Delta t]^2/2| \leq 2 - \alpha\Delta t + [\Delta t]^2/2. \tag{4.7}$$

Condition (4.6) is equivalent to the following sequence of inequalities:

$$-1 \leq 1 - \alpha\Delta t + [\Delta t]^2/2 \leq 1, \tag{4.8}$$

$$\left.\begin{array}{r} [\Delta t]^2/2 \leq \alpha\Delta t, \\ 2 - \alpha\Delta t + [\Delta t]^2/2 \geq 0, \end{array}\right\} \tag{4.9}$$

$$\left.\begin{array}{ll} \text{(a)} & \Delta t \leq 2\alpha, \\ \text{(b)} & 2 - \alpha\Delta t + [\Delta t]^2/2 \geq 0. \end{array}\right\} \tag{4.10}$$

Condition (4.7) is equivalent to

$$\left.\begin{array}{r} -2 + \alpha\Delta t + [\Delta t]^2/2 \leq 2 - \alpha\Delta t + [\Delta t]^2/2, \\ 2 - \alpha\Delta t - [\Delta t]^2/2 \leq 2 - \alpha\Delta t + [\Delta t]^2/2, \end{array}\right\} \tag{4.11}$$

which is equivalent to

$$\Delta t \leq 2/\alpha. \tag{4.12}$$

Since (4.12) implies (4.10b), we have proved that (4.5a) is equivalent to

$$\left.\begin{array}{ll} \text{(a)} & \Delta t \leq 2\alpha, \\ \text{(b)} & \Delta t \leq 2/\alpha. \end{array}\right\} \tag{4.13}$$

To complete the proof of Theorem 4.1, we examine condition (4.5b) under the assumption that (4.5a) and (4.13) hold.

Then $|\lambda_1| = |\lambda_2| = 1$ iff

$$\begin{aligned} |\lambda_1 \lambda_2| &= 1, \\ &= |1 - \alpha\Delta t(1 - \Delta t/2\alpha)|, \\ &= |(1 + [\Delta t]^2/2) - \alpha\Delta t|. \end{aligned}$$

Noting (4.13), it follows that $|\lambda_1| = |\lambda_2| = 1$ iff

$$\Delta t = 2\alpha. \tag{4.14}$$

If (4.14) holds, (4.3) simplifies to

$$\lambda^2 + \lambda(-2 + 4\alpha^2) + 1. \tag{4.15}$$

Remembering that $\alpha > 0$, it is easily proved that the zeros of (4.15) are equal iff $\alpha = 1$. We have thus proved that (4.5b) is equivalent to the statement,

$$\text{If} \quad \Delta t = 2\alpha, \quad \text{then} \quad \alpha \neq 1. \tag{4.16}$$

Combining (4.13) and (4.16), we obtain (4.2).

5. Discrete mechanics: stability of the nonlinear model

We repeat equations (3.2):

$$x_{n+1} = x_n + \Delta t(v_n + v_{n+1})/2, \tag{5.1}$$

$$v_{n+1} = v_n - \Delta t(\alpha v_n + \sin x_n). \tag{5.2}$$

The following theorem gives sufficient conditions for the stability of the scheme (5.1), (5.2):

Theorem 5.1.

If x_n, v_n, is the solution of (5.1), (5.2), and

$$\Delta t < \min\{2\alpha,\ 2/\alpha\}, \tag{5.3}$$

then, as $n \to \infty$,

$$v_n \to 0, \tag{5.4}$$

and

$$x_n \to k\pi, \tag{5.5}$$

where k is some integer.

Proof:

The proof is a modification of the proof of Theorem 2.1 and also uses some of the ideas connected with the "energy method" (Richtmyer and Morton [6, p. 132]).

Multiplying (5.2) by $(v_{n+1} + v_n)/2$, and using (5.1),

$$(v_{n+1}^2 - v_n^2)/2 = -(v_{n+1} + v_n)\Delta t(\alpha v_n + \sin x_n)/2.$$

$$= -\alpha\Delta t(v_{n+1} + v_n)v_n/2 - (x_{n+1} - x_n)\sin x_n. \tag{*}$$

Applying Taylor's series,

$$-(\cos x_{n+1} - \cos x_n) = +(x_{n+1} - x_n)\sin x_n + [(x_{n+1} - x_n)^2 \cos \tilde{x}_n]/2,$$

where $\tilde{x}_n$ lies between x_n and x_{n+1}. Hence,

$$-(\cos x_{n+1} - \cos x_n) \leq (x_{n+1} - x_n)\sin x_n + (x_{n+1} - x_n)^2/2. \tag{**}$$

Adding (*) and (**), and using (5.1),

$$(v_{n+1}^2 - v_n^2)/2 - (\cos x_{n+1} - \cos x_n)$$

$$\leq -\alpha\Delta t(v_{n+1} + v_n)v_n/2 + (\Delta t)^2(v_{n+1} + v_n)^2/8,$$

$$= \frac{\Delta t}{8}[\Delta t v_{n+1}^2 + (\Delta t - 4\alpha)v_n^2 - (4\alpha - 2\Delta t)v_n v_{n+1}],$$

$$= \frac{\Delta t}{8}[-(2\alpha - \Delta t)(v_{n+1} + v_n)^2 + 2\alpha(v_{n+1}^2 - v_n^2)].$$

Hence,

$$E_{n+1} - E_n \leq -\Delta t(2\alpha - \Delta t)(v_{n+1} + v_n)^2/8, \tag{5.6}$$

where,

$$E_n = v_n^2(1 - \alpha\Delta t/2)/2 - \cos x_n. \tag{5.7}$$

Summing (5.6), we find that

$$E_n + F_n \leq E_0, \tag{5.8}$$

where

$$F_n = \frac{\Delta t}{8}(2\alpha - \Delta t)\sum_{k=1}^{n}(v_k + v_{k-1})^2. \tag{5.9}$$

Up to this point the proof has parallelled the proof of Theorem 2.1, equations (5.6) through (5.9) being discrete analogs of equations (2.6) through (2.9). We have made no use of the stability condition (5.3). In order to continue the proof we now assume that (5.3) holds so that $2\alpha - \Delta t > 0$ and $1 - \alpha\Delta t/2 > 0$. We remark in passing that this is the usual way in which stability conditions arise in the "energy method" (see Richtmyer and Morton [6, p. 133]).

As in Theorem 2.1, several important conclusions follow from (5.8):

$$|v_n| \leq V = \frac{2(1 + E_0)^{1/2}}{1 - \alpha\Delta t/2} , \tag{5.10}$$

$$F_n \uparrow L, \tag{5.11}$$

$$|x_{n'} - x_{n''}| \leq V\Delta t|n' - n''|. \tag{5.12}$$

From this point, the proof differs from the proof of Theorem 2.1, the reason being that (2.8) is an equality while (5.8) is an inequality.

From (5.9) and (5.11) it follows that

$$v_{n+1} + v_n \to 0, \tag{5.13}$$

so that, from (5.1),

$$x_{n+1} - x_n \to 0. \tag{5.14}$$

Adding to (5.2) the equation obtained from (5.2) by replacing n by $n+1$, and using (5.13),

$$\sin x_n + \sin x_{n+1} \to 0. \tag{5.15}$$

It is easily seen from (5.14) and (5.15) that

$$\sin x_n \to 0, \tag{5.16}$$

and that

$$x_n \to k\pi, \tag{5.17}$$

for some integer k. We have thus proved (5.5).

To prove (5.4) it suffices to observe that (5.2) may be rewritten in the form

$$v_{n+1} + v_n = (2 - \alpha\Delta t)v_n - \Delta t \sin x_n.$$

Appendix

We prove that if λ_1 and λ_2 are the zeros of the real quadratic equation

$$\lambda^2 + 2b\lambda + c = 0, \tag{A.1}$$

then a necessary and sufficient condition that

$$|\lambda_1| \leq 1 \quad \text{and} \quad |\lambda_2| \leq 1, \tag{A.2}$$

is that

$$|c| \leq 1, \tag{A.3}$$

and

$$|b| \leq (1 + c)/2. \tag{A.4}$$

There are two cases to consider:

Case 1: $b^2 < c$.

Necessity: Assume that (A.2) holds. Since $c = \lambda_1 \lambda_2$, (A.3) must hold. Also,

$$\begin{aligned} b^2 < c &= [2c + 2c]/4, \\ &\leq [2c + 1 + c^2]/4, \\ &= (1 + c)^2/4, \end{aligned}$$

from which (A.4) follows.

Sufficiency: Assume that (A.3) and (A.4) hold. Since λ_1 and λ_2 are both complex, $|\lambda_1|^2 = |\lambda_2|^2 = |c|$, so that (A.2) holds.

Case 2: $b^2 \geq c$.

Then λ_1 and λ_2 are both real. Hence (A.2) is equivalent to each of the following systems of inequalities:

$$-1 \le -b - (b^2 - c)^{1/2} \le -b + (b^2 - c)^{1/2} \le 1, \quad \text{(A.5)}$$

$$\left.\begin{aligned} -b + (b^2 - c)^{1/2} &\le 1, \\ +b + (b^2 - c)^{1/2} &\le 1, \end{aligned}\right\} \quad \text{(A.6)}$$

$$|b| + (b^2 - c)^{1/2} \le 1, \quad \text{(A.7)}$$

$$\left.\begin{aligned} (b^2 - c)^{1/2} &\le 1 - |b|, \\ |b| &\le 1, \end{aligned}\right\} \quad \text{(A.8)}$$

$$\left.\begin{aligned} |b|^2 - c &\le 1 - 2|b| + |b|^2, \\ |b| &\le 1, \end{aligned}\right\} \quad \text{(A.9)}$$

$$\left.\begin{aligned} |b| &\le (1 + c)/2, \\ |b| &\le 1, \end{aligned}\right\} \quad \text{(A.10)}$$

$$\left.\begin{aligned} |b| &\le (1 + c)/2, \\ |c| &\le 1. \end{aligned}\right\} \quad \text{(A.11)}$$

Therefore, (A.2) is equivalent to (A.3) and (A.4).

Since (A.1) is only a quadratic, it was possible to derive (A.3) and (A.4) directly. When analyzing the stability of more complicated problems, (A.1) is replaced by

$$\sum_{k=1}^{n} a_k \lambda^k = 0, \quad \text{(A.12)}$$

while (A.2) is replaced by,

$$|\lambda_k| \leq 1, \quad \text{for} \quad 1 \leq k \leq n. \tag{A.13}$$

For such problems, when determining conditions upon the coefficients a_k which are equivalent to (A.13), it is necessary to use the general theorems of Hurwitz or Schur-Cohn (see Marden [5, p. 194]).

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13. ABSTRACT

Greenspan has developed a discrete version of Newtonian mechanics, called discrete mechanics, which he has used to derive finite difference approximations to nonlinear problems in mechanics. The stability of one of these finite difference approximations is analysed in detail using the "energy method". It is found that the theoretical stability limit agrees exactly with the stability limit observed experimentally.

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