NUMERICAL STUDIES OF PROTOTYPE CAVITY FLOW PROBLEMS

by Donald Greenspan

APPENDIX:
CDC 3600 FORTRAN PROGRAM FOR CAVITY FLOW PROBLEMS

by D. Schultz

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1. Introduction.

The flow of a gas or of a liquid in a closed cavity has long been of interest in applied science (see, e.g., references [1, 2, 4, 7-12, 14] and the additional references contained therein). In this paper we will apply the power of the high speed digital computer to study prototype, steady state, two dimensional problems for such flows. The numerical methods to be developed will be finite difference methods and will be described in sufficient generality so as to be applicable to nonlinear coupled systems similar in structure to the Navier-Stokes equations.

2. The Eddy Problem in a Rectangle.

The class of problems to be studied, called eddy problems in a rectangle, can be formulated as follows. For $d > 0$, let the points $(0, 0)$, $(1, 0)$, $(1, d)$ and $(0, d)$ be denoted by A, B, C and D, respectively (see Figure 2.1). Let S be the rectangle whose vertices are A, B, C, D and denote its interior by R. On R the equations of motion to be satisfied are the Navier-Stokes equations, that is

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$$\Delta\psi = -\omega \tag{2.1}$$

$$\Delta\omega + \Re\left(\frac{\partial\psi}{\partial x}\frac{\partial\omega}{\partial y} - \frac{\partial\psi}{\partial y}\frac{\partial\omega}{\partial x}\right) = 0 , \tag{2.2}$$

where ψ is the stream function, ω is the vorticity and $\Re$ is the Reynolds number. On S the boundary conditions to be satisfied are

$$\psi = 0, \quad \frac{\partial\psi}{\partial x} = 0 , \quad \text{on AD} \tag{2.3}$$

$$\psi = 0, \quad \frac{\partial\psi}{\partial y} = 0 , \quad \text{on AB} \tag{2.4}$$

$$\psi = 0, \quad \frac{\partial\psi}{\partial x} = 0 , \quad \text{on BC} \tag{2.5}$$

$$\psi = 0, \quad \frac{\partial\psi}{\partial y} = -1 , \quad \text{on CD} . \tag{2.6}$$

The analytical problem is defined on R + S by (2.1)-(2.6) and is shown diagrammatically in Figure 2.1.

In general, boundary value problem (2.1)-(2.6) cannot be solved by means of existing analytical techniques. Physical solutions have been produced in the laboratory by Pan and Acrivos [9], while numerical methods which "converge", but only for small $\Re$, have been developed by Burggraf [4] and Runchal, Spalding and Wolfshtein [12]. A numerical method which converges for all $\Re$, but which has been run only for relatively large values of the grid size, has been developed by the writer [7] .

We shall describe next a modified, somewhat faster form of the method developed in [7] and apply it to a selection of difficult problems which are of wide interest. Among our major objectives will be the construction of secondary vortices and the study of vorticity for large Reynolds number.

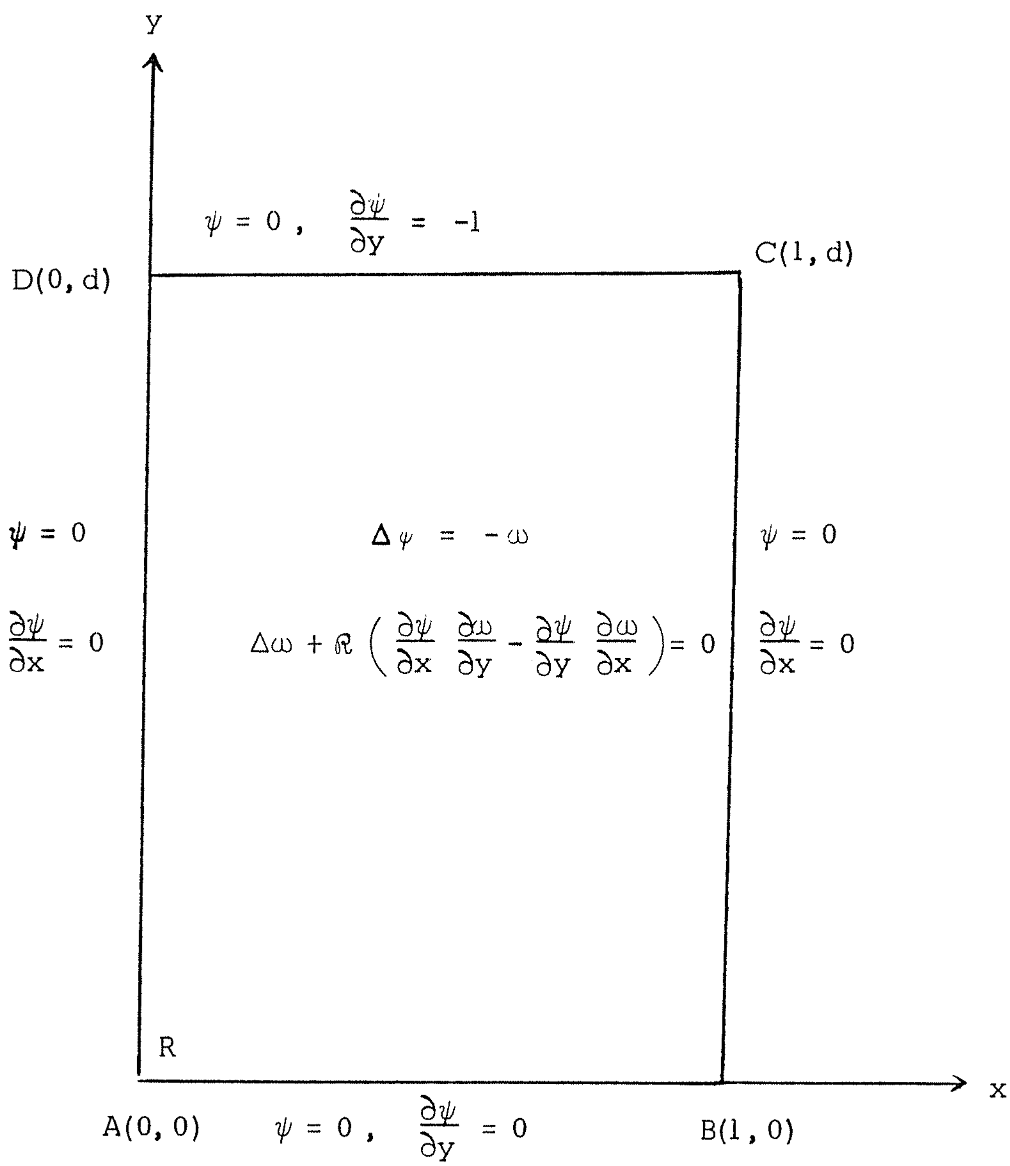

Figure 2.1

3. The General Numerical Method.

For a fixed positive integer n, set $h = \frac{1}{n}$. Assume, for simplicity, that d is an integral multiple of h. (If d is not an integral multiple of h, the method is easily modified as shown in [7].) Starting at $(0, 0)$ with grid size h, construct and number in the usual way [7] the set of interior grid points R_h and the set of boundary grid points S_h.

For given tolerances ε_1 and ε_2, we will show first how to construct on R_h a sequence of discrete stream functions

$$\psi^{(0)},\ \psi^{(1)},\ \psi^{(2)},\ \cdots \tag{3.1}$$

and on $R_h + S_h$ a sequence of discrete vorticity functions

$$\omega^{(0)},\ \omega^{(1)},\ \omega^{(2)},\ \cdots\ , \tag{3.2}$$

such that for some integer k both the following are valid:

$$|\psi^{(k)} - \psi^{(k+1)}| < \varepsilon_1\ , \quad \text{on } R_h \tag{3.3}$$

$$|\omega^{(k)} - \omega^{(k+1)}| < \varepsilon_2\ , \quad \text{on } R_h + S_h\ . \tag{3.4}$$

Initially, set

$$\psi^{(0)} = C_1\ , \quad \text{on } R_h \tag{3.5}$$

$$\omega^{(0)} = C_2\ , \quad \text{on } R_h + S_h\ , \tag{3.6}$$

where C_1 and C_2 are constants.

To produce the second iterate $\psi^{(1)}$ of sequence (3.1) proceed as follows. At each point of R_h of the form (h, ih), $i = 2, \cdots, n-2$, approximate (2.3) by

$$(3.7) \qquad \psi(h, ih) = \frac{\psi(2h, ih)}{4} .$$

At each point of R_h of the form (ih, h), $i = 1, 2, \cdots, n-1$, approximate (2.4) by

$$(3.8) \qquad \psi(ih, h) = \frac{\psi(ih, 2h)}{4} .$$

At each point of R_h of the form $(1-h, ih)$, $i = 2, 3, \cdots, n-2$, approximate (2.5) by

$$(3.9) \qquad \psi(1-h, ih) = \frac{\psi(1-2h, ih)}{4} .$$

At each point of R_h of the form $(ih, 1-h)$, $i = 1, 2, \cdots, n-1$, approximate (2.6) by

$$(3.10) \qquad \psi(ih, 1-h) = \frac{h}{2} + \frac{\psi(ih, 1-2h)}{4} .$$

And at each remaining point of R_h write down the difference analogue

$$(3.11) \qquad -4\psi(x, y) + \psi(x+h, y) + \psi(x, y+h) + \psi(x-h, y) + \psi(x, y-h) = -h^2 \omega^{(0)}(x, y)$$

of (2.1). Solve the linear algebraic system generated by (3.7)-(3.11) by the generalized Newton's method [7] with over-relaxation factor r_ψ and denote this solution by $\overline{\psi}^{(1)}$. Then, on R_h, $\psi^{(1)}$ is defined by the smoothing formula

$$\psi^{(1)} = \rho\,\psi^{(0)} + (1-\rho)\,\overline{\psi}^{(1)}, \quad 0 \le \rho \le 1 . \tag{3.12}$$

To produce the second iterate $\omega^{(1)}$ of sequence (3.2) proceed as follows. At each point of S_h of the form $(ih, 0)$, $i = 0, 1, 2, \cdots, n$, set

$$\overline{\omega}^{(1)}(ih, 0) = -\frac{2\psi^{(1)}(ih, h)}{h^2} ; \tag{3.13}$$

at each point of S_h of the form $(0, ih)$, $i = 1, 2, \cdots, n-1$, set

$$\overline{\omega}^{(1)}(0, ih) = -\frac{2\psi^{(1)}(h, ih)}{h^2} ; \tag{3.14}$$

at each point of S_h of the form $(1, ih)$, $i = 1, 2, \cdots, n-1$, set

$$\overline{\omega}^{(1)}(1, ih) = -\frac{2\psi^{(1)}(1-h, ih)}{h^2} ; \tag{3.15}$$

and at each point of S_h of the form $(ih, 1)$, $i = 0, 1, 2, \cdots, n$, set

$$\overline{\omega}^{(1)}(ih, 1) = \frac{2}{h} - \frac{2\psi^{(1)}(ih, 1-h)}{h^2} . \tag{3.16}$$

Next, at each point (x, y) in R_h set

$$\alpha = \psi^{(1)}(x+h, y) - \psi^{(1)}(x-h, y)$$

$$\beta = \psi^{(1)}(x, y+h) - \psi^{(1)}(x, y-h)$$

and approximate (2.2), appropriately, by

$$\left(-4 - \frac{\alpha\mathfrak{R}}{2} - \frac{\beta\mathfrak{R}}{2}\right)\omega(x, y) + \omega(x+h, y) + \left(1 + \frac{\alpha\mathfrak{R}}{2}\right)\omega(x, y+h) + \left(1 + \frac{\beta\mathfrak{R}}{2}\right)\omega(x-h, y) + \omega(x, y-h) = 0; \quad \text{if} \quad \alpha \ge 0, \quad \beta \ge 0 , \tag{3.17}$$

(3.18) $$(-4 - \frac{\alpha\Re}{2} + \frac{\beta\Re}{2})\omega(x, y) + (1 - \frac{\beta\Re}{2})\omega(x+h, y) + (1 + \frac{\alpha\Re}{2})\omega(x, y+h)$$

$$+ \omega(x-h, y) + \omega(x, y-h) = 0 ; \quad \text{if } \alpha \geq 0, \quad \beta < 0 ,$$

(3.19) $$(-4 + \frac{\alpha\Re}{2} - \frac{\beta\Re}{2})\omega(x, y) + \omega(x+h, y) + \omega(x, y+h) + (1 + \frac{\beta\Re}{2})\omega(x-h, y)$$

$$+ (1 - \frac{\alpha\Re}{2})\omega(x, y-h) = 0 ; \quad \text{if } \alpha < 0, \quad \beta \geq 0 ,$$

(3.20) $$(-4 + \frac{\alpha\Re}{2} + \frac{\beta\Re}{2})\omega(x, y) + (1 - \frac{\beta\Re}{2})\omega(x+h, y) + \omega(x, y+h)$$

$$+ \omega(x-h, y) + (1 - \frac{\alpha\Re}{2})\omega(x, y-h) = 0 ; \quad \text{if } \alpha < 0, \quad \beta < 0 .$$

Solve the linear algebraic system generated by (3.17)-(3.20) by the generalized Newton's method with over-relaxation factor r_ω and denote the solution by $\overline{\omega}^{(1)}$. Finally, on all of $R_h + S_h$ define $\omega^{(1)}$ by the smoothing formula

$$\omega^{(1)} = \mu\omega^{(0)} + (1 - \mu)\overline{\omega}^{(1)} , \quad 0 \leq \mu \leq 1 .$$

Proceed next to determine $\psi^{(2)}$ on R_h from $\omega^{(1)}$ and $\psi^{(1)}$ in the same fashion as $\psi^{(1)}$ was determined from $\omega^{(0)}$ and $\psi^{(0)}$. Then construct $\omega^{(2)}$ on $R_h + S_h$ from $\omega^{(1)}$ and $\psi^{(2)}$ in the same fashion as $\omega^{(1)}$ was determined from $\omega^{(0)}$ and $\psi^{(1)}$. In the indicated fashion, construct the sequences (3.1) and (3.2). Terminate the computation when (3.3) and (3.4) are valid.

Finally, when $\psi^{(k)}$ and $\omega^{(k)}$ are verified to be solutions of the difference analogues of (2.1) and (2.2), they are taken to be the numerical approximations of $\psi(x, y)$ and $\omega(x, y)$, respectively.

4. Examples.

Consider first the boundary value problem defined by (2.1)-(2.6) with $d = 1$. This problem was solved by the method of Section 3 for $\Re = 200$ with $h = \frac{1}{20}$, $\varepsilon_1 = 1$, $\varepsilon_2 = 10^{-4}$, $\rho = 0.1$, $\mu = 0.7$, $r_\psi = 1.8$, $r_\omega = 1.0$, $C_1 = C_2 = 0$, and also for $\Re = 500$, 2000 and 15000 with the same parameter values except for $\varepsilon_2 = 10^{-3}$. Convergence was achieved for $\Re = 200$ in 14 minutes with 341 outer iterations, for $\Re = 500$ in 11 minutes with 96 outer iterations, for $\Re = 2000$ in 4 minutes with 80 outer iterations, and for $\Re = 15000$ in $3\frac{1}{2}$ minutes with 40 outer iterations. The resulting stream curves exhibited only primary vortices and are shown in Figure 4.1. The resulting equivorticity curves exhibited the double spiral development shown in [7] and are given in Figure 4.2.

With an aim toward producing secondary vortices and toward studying vorticity for large Reynolds numbers, boundary value problems (2.1)-(2.6) was considered again with $d = 1$. The problem was solved for $\Re = 50$, 10000, and 100000 with $h = \frac{1}{40}$. For $\Re = 50$ the remaining input parameters were chosen to be $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-3}$, $\rho = .03$, $\mu = .90$, $r_\psi = 1.8$, $r_\omega = 1.8$, $C_1 = C_2 = 0$. Convergence was achieved in 60 minutes with 100 outer iterations. The resulting flow with the secondary vortices is shown in Figure 4.3. For $\Re = 10000$ the remaining input parameters were chosen to be $\varepsilon_1 = .004$, $\varepsilon_2 = .03$, $\rho = .03$, $\mu = .95$, $r_\psi = 1.8$, $r_\omega = 1$, $C_1 = C_2 = 0$. After 183 outer iterations, μ was changed to .85. Convergence was achieved in 260 minutes with a total of 226 outer iterations. The resulting flow with a single secondary vortex is shown in Figure 4.4. For $\Re = 100000$, the remaining input parameters were chosen to be $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = .005$, $\rho = .03$, $\mu = .95$, $r_\psi = 1.8$, $r_\omega = 1$, but $\psi^{(0)}$ and $\omega^{(0)}$ were taken to be the 57^{th} outer iterates of the run for $\Re = 10000$. Convergence was achieved in 135 minutes with 386 outer iterations. The flow is shown in Figure 4.5 and contains no secondary vortices. The equivorticity curve $\omega = 1.630$, with its double-spiral, space filling characteristics is shown in Figure 4.6. Numerical evidence of Batchelor's result that the vorticity in a large subregion of R converges to a constant as $R \to \infty$ is exhibited in Figure 4.6 by setting crosses on those points at which the vorticity is between 1.6 and 1.7 .

Finally, consider boundary value problem (2.1)-(2.6) with $d = 2$ and $R = 10$. This problem was solved with $h = \frac{1}{40}$, $\varepsilon_1 = 10^{-4}$, $\varepsilon_2 = 10^{-3}$, $\rho = .05$, $\mu = .85$, $r_\psi = 1.8$, $r_\omega = 1.25$, $C_1 = C_2 = 0$. Convergence was achieved in 32 minutes with 102 outer iterations. The resulting flow, with its two primary and two secondary vortices, is shown in Figure 4.7.

5. <u>Remarks</u>.

From the many examples run in addition to those described in Section 4, the following observations and heuristic conclusions resulted. Divergence or exceptionally slow convergence usually followed if any one of the following choices were made: $.4 \le \rho \le 1$, $0 \le \mu \le .6$, $r_\psi < 1$, $r_\omega << 1$. The choice $\rho = \mu = 0$ yields convergence only for large grid sizes and small Reynolds numbers. The choice $r_\psi = 1.8$ was consistently good. For grid sizes larger than or equal to $\frac{1}{20}$, sequence (3.1) converged so much faster than (3.2) that very little attention had to be directed toward the choice of ε_1, but for grids smaller than $\frac{1}{20}$ this was not the case and attention had to be directed to the choices of both ε_1 and ε_2. Deletion of all or even of some of the special formulas (3.7)-(3.10) and substitution with (3.11) always led to divergence for large Reynolds numbers ($\Re \sim 10000$), but often did yield secondary vortices for $h = \frac{1}{20}$ for small Reynolds numbers ($\Re \sim 50$). The difference equations for $\psi^{(k)}$ and $\omega^{(k)}$ were always satisfied to much smaller tolerances than those imposed in (3.3) and (3.4), respectively.

Several possible modifications of the method of this paper which should be explored if one wishes to speed up the convergence include allowing some or all of ρ, μ, r_ψ and r_ω to be variable [6], using line over relaxation [15], and choosing $\psi^{(0)}$ and $\omega^{(0)}$ in a more judicious manner than that prescribed in (3.5)-(3.6).

Observe also that the method of Section 3 applies directly to biharmonic problems (i.e., to the case $\Re = 0$) and initial computations verify that it extends in a natural way to free convection problems [1].

Finally, note that theoretical support for the method of this paper is now beginning to appear for very special cases [3, 5, 13].

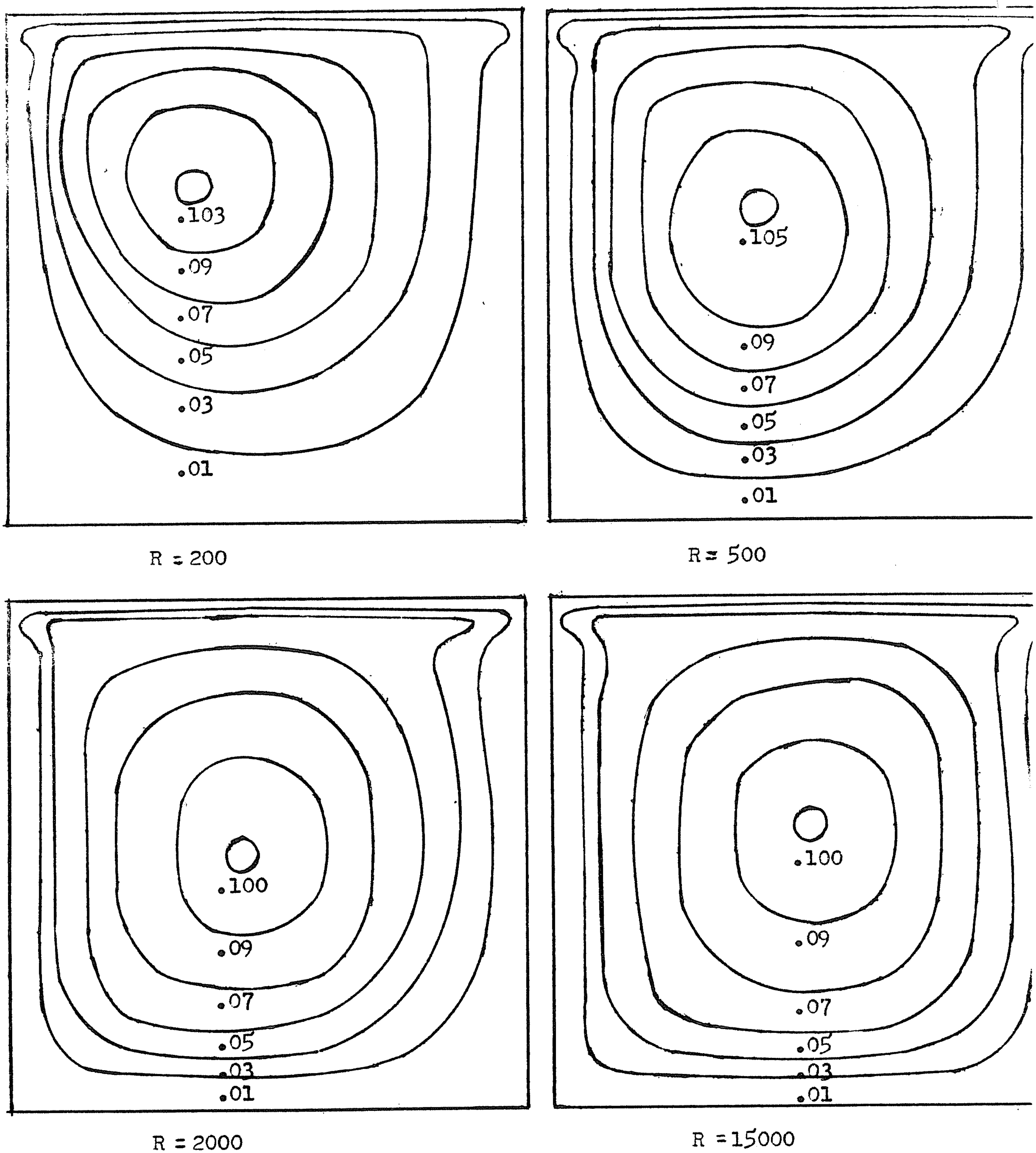

FIGURE 4.1 Typical streamlines for h = 1/20.

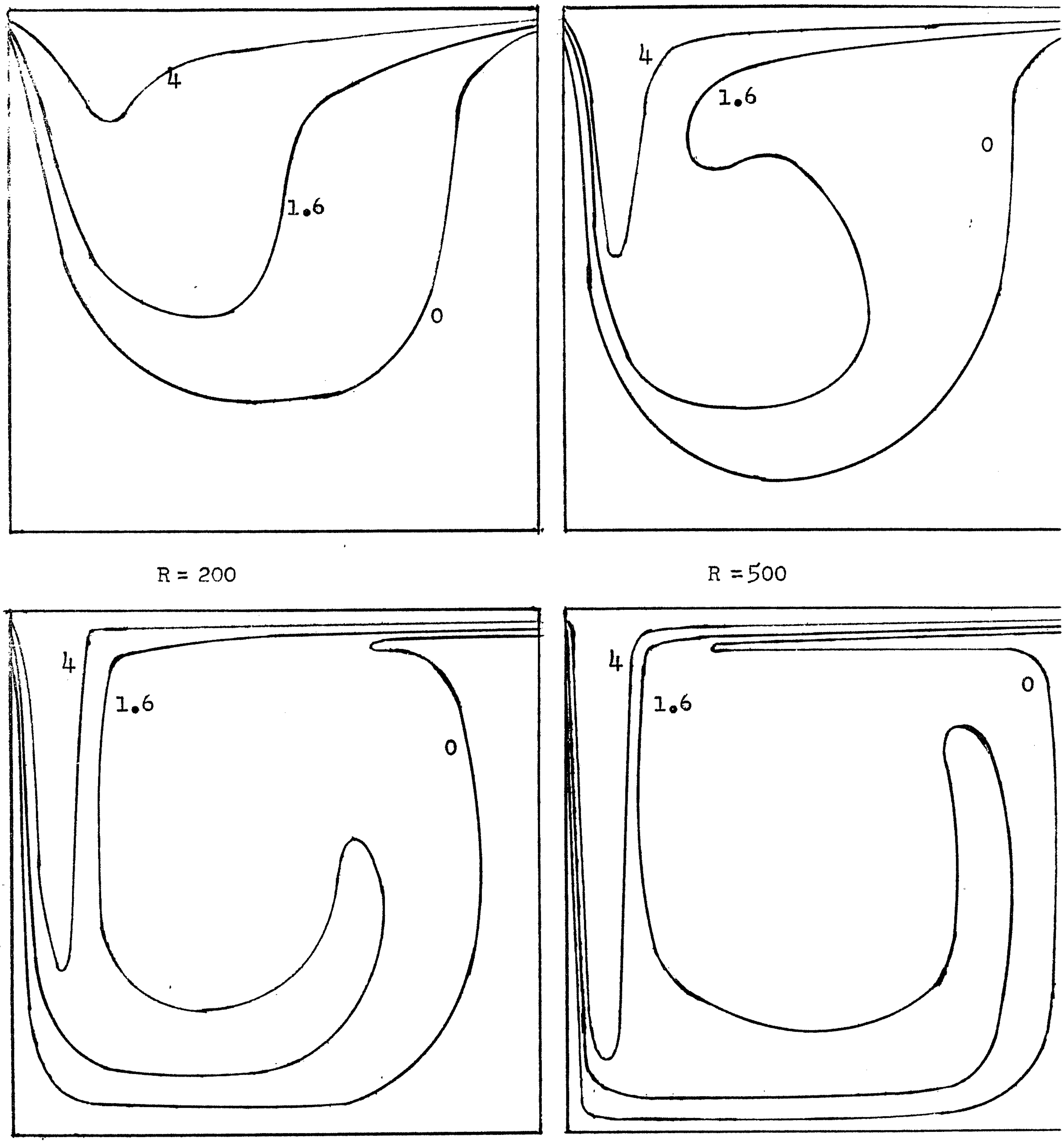

FIGURE 4.2 Selected equivorticity curves for h = 1/20.

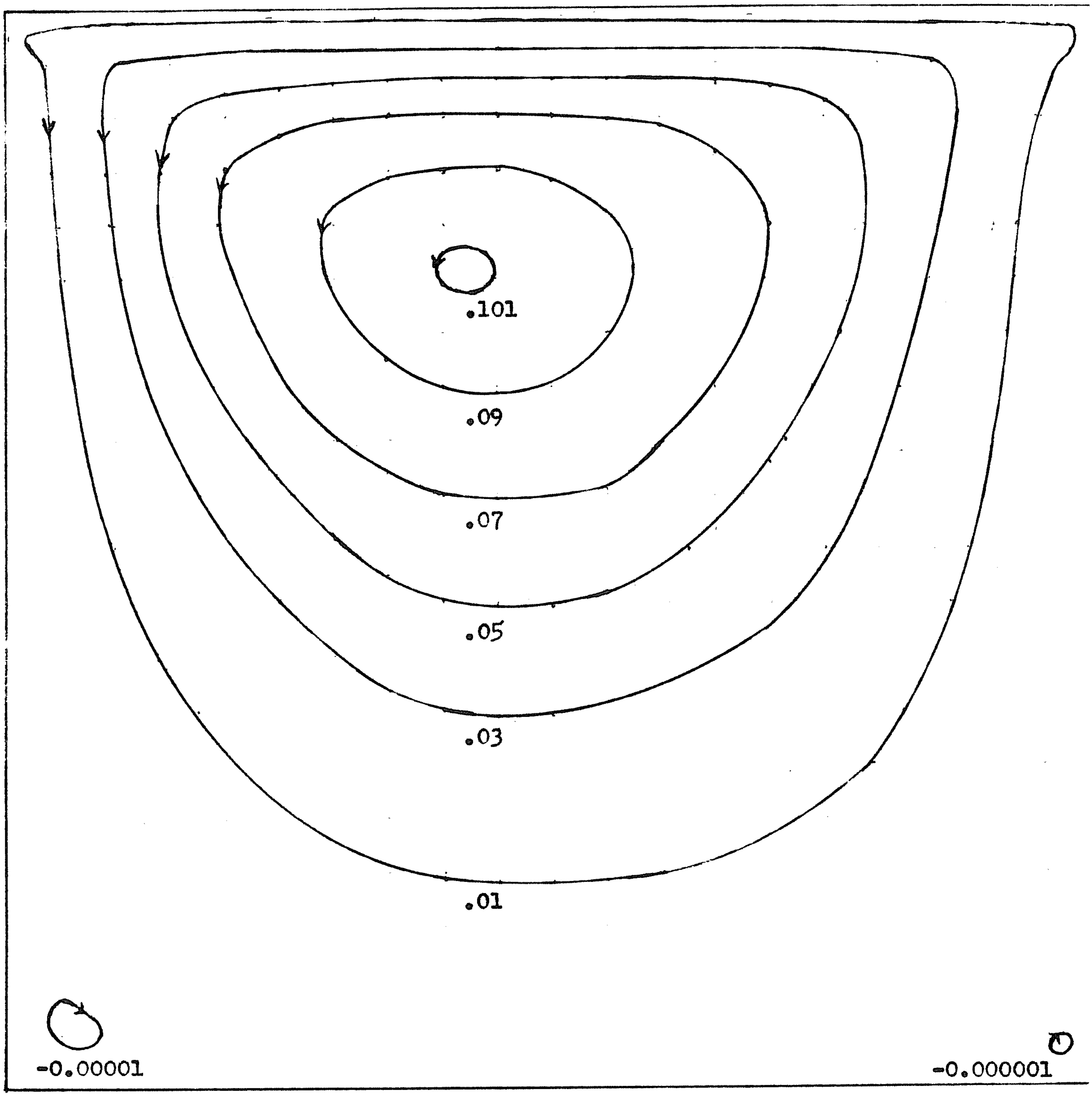

FIGURE 4.3 Streamlines for Reynolds number 50 with h = 1/40.

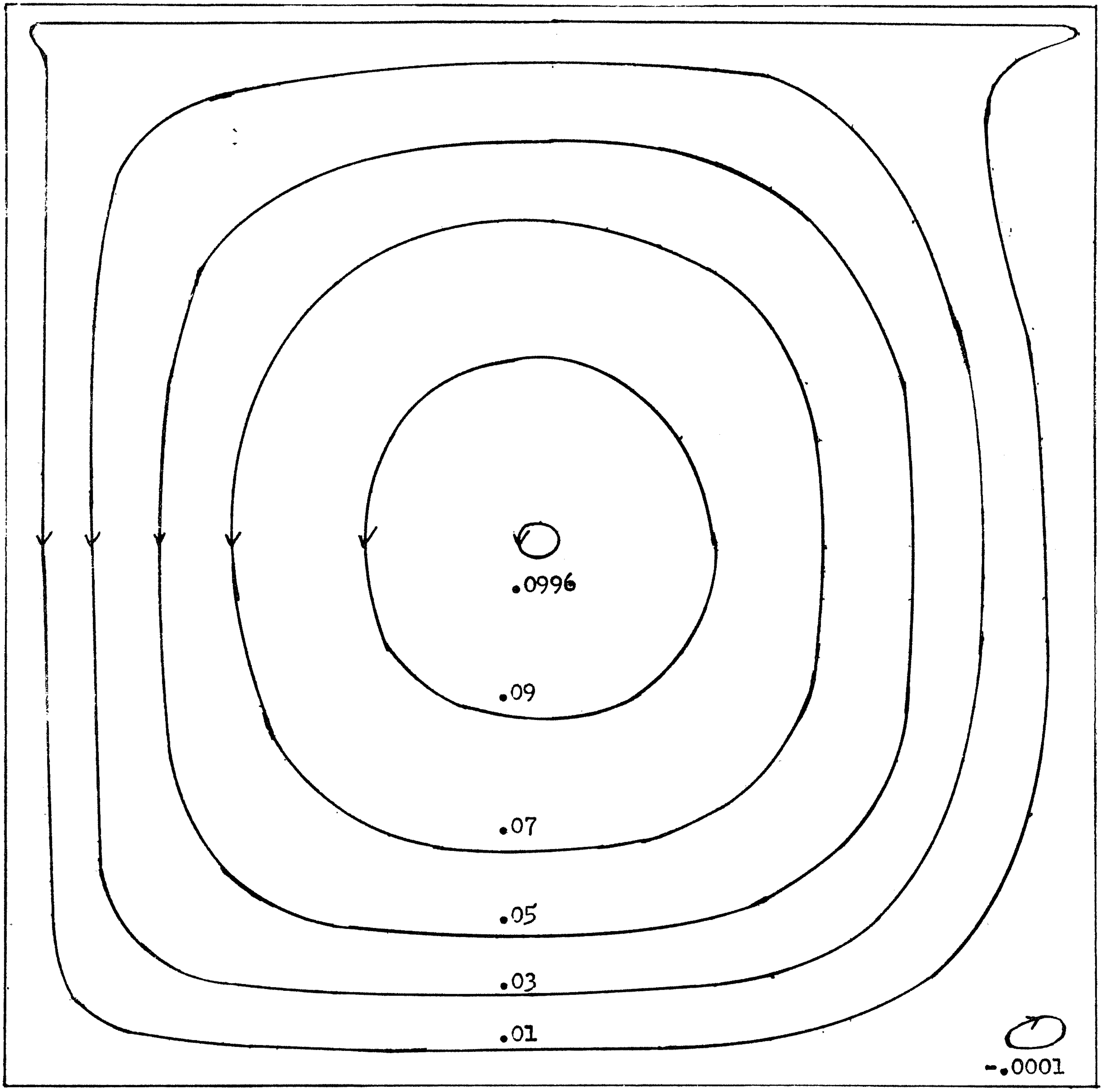

FIGURE 4.4 Streamlines for Reynolds number 10000 with h = 1/40.

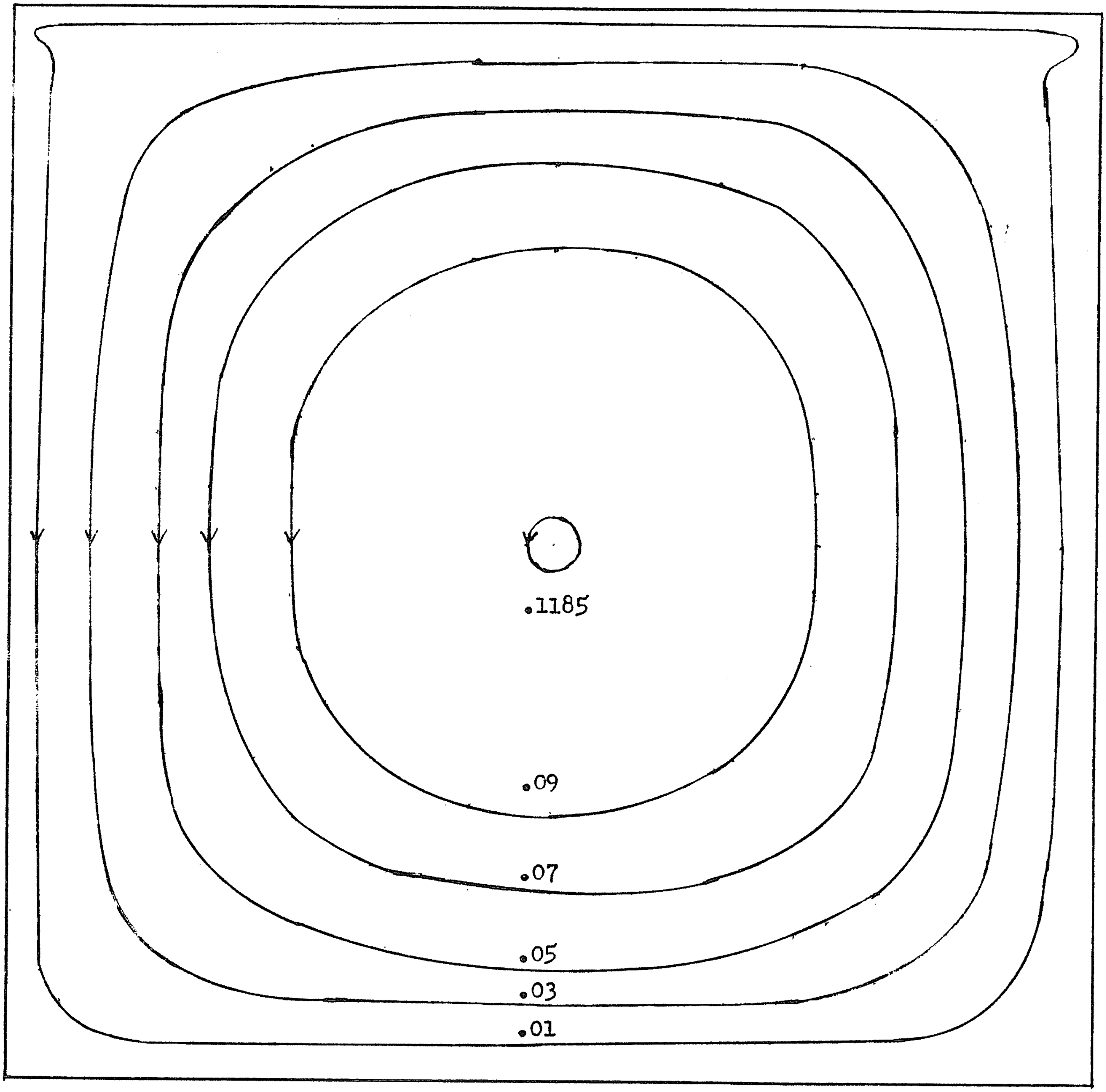

FIGURE 4.5 Streamlines for Reynolds number 100000 with $h = 1/40$.

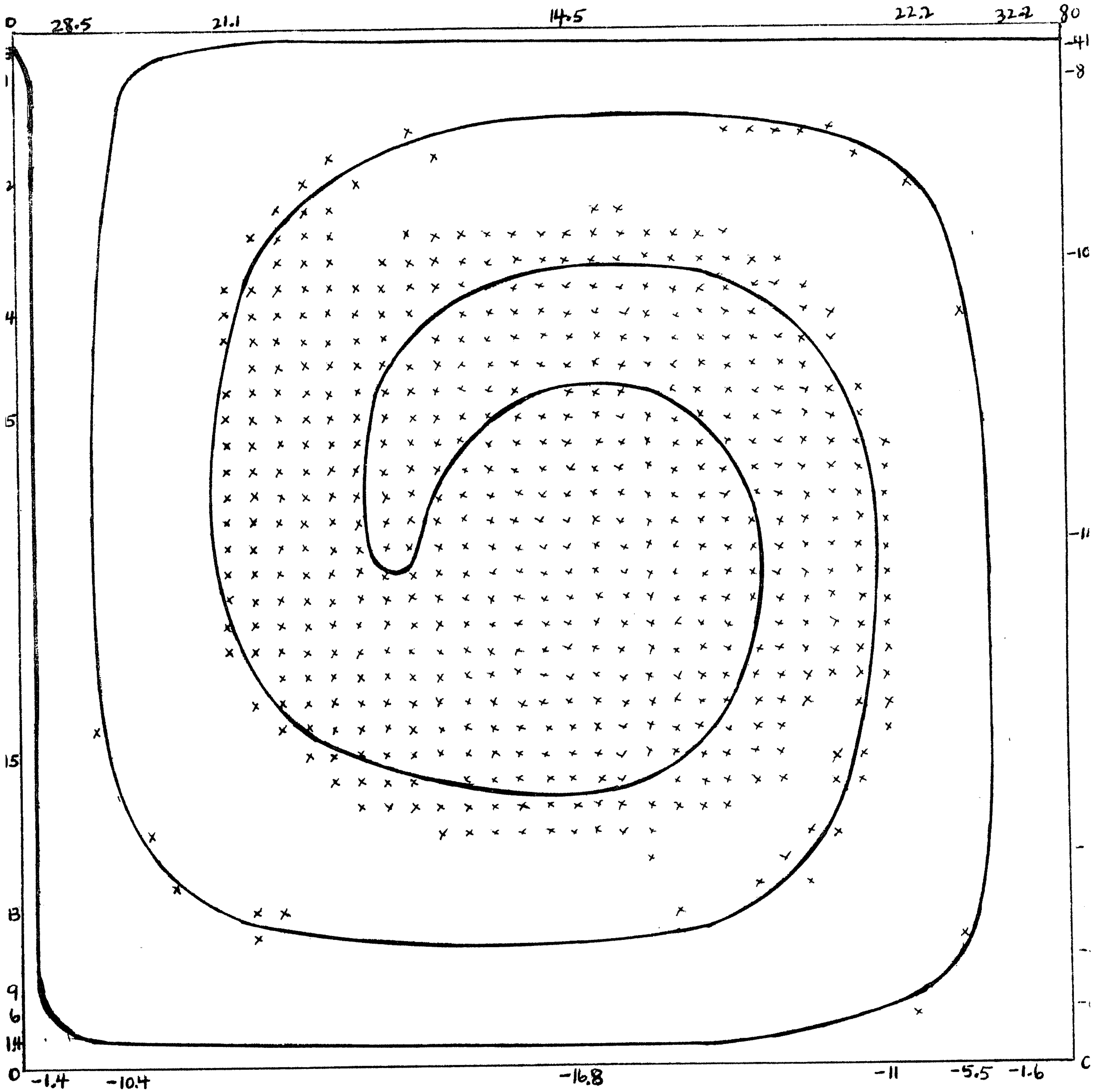

FIGURE 4.6 Equivorticity curve $\omega = 1.630$ for Reynolds number 100000 and $h = 1/40$. At crossed points vorticity is between 1.6 and 1.7.

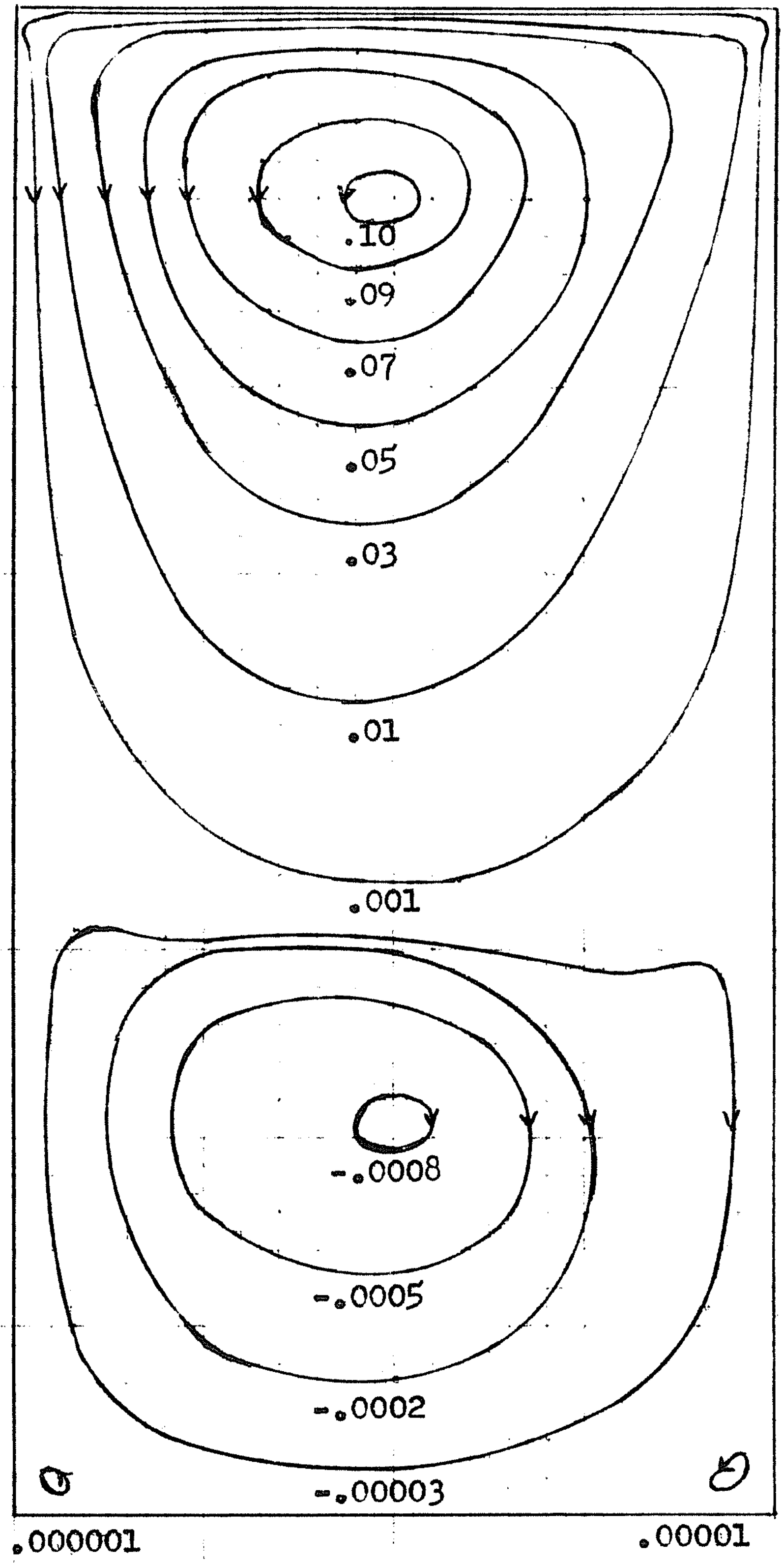

FIGURE 4.7 Streamlines for Reynolds number 10 with h = 1/40 for a 2 by 1 rectangular cavity.

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APPENDIX

CDC 3600 FORTRAN PROGRAM FOR CAVITY FLOW PROBLEMS

D. SCHULTZ

DEFINITIONS OF PROGRAM VARIABLES

OMA = VORTICITY VALUES

PSI = STREAM VALUES

N = NUMBER OF VERTICAL SPACES IN THE GRID

M = NUMBER OF HORIZONTAL SPACES IN THE GRID

R = REYNOLD'S NUMBER

H = GRID SIZE

EPS = TOLERANCE FOR INNER-AND OUTER-ITERATIONS

C1 = WEIGHTING FACTOR FOR OMA

F1 = WEIGHTING FACTOR FOR PSI

RW = RELAXATION FACTOR FOR OMA EQUATIONS

NM = NUMBER OF OUTER-ITERATIONS

NCOUNT = NUMBER OF INNER-ITERATIONS

W0,W1,W2,W3,W4 = COEFFICIENTS FOR THE VORTICITY EQUATION

ISTOP = SWITCH TO INDICATE CONVERGENCE

```
      PROGRAM NAVSTK
      DIMENSION PSI(50,50),OMA(50,50),SVPSI(50,50),SVOMA(50,50),SVOUT(5
     10,50)
      COMMON N,NPLUS1,M,MPLUS1,NZ,MP
      READ 300,N,M
300   FORMAT(2I2)
      MPLUS1=M+1
      MMESH=M-1
      JM=0
      NPLUS1=N+1
      NMESH=N-1
      H=1./N
      H2=H*H
      EPS=.001
C     INITIALIZE VECTORS
      NZ=0
      MP=5
      ISTOP=0
      R=50
      RW=1.
      C1=0
      F1=0
104   CONTINUE
      PRINT 2323,C1
2323  FORMAT(1H1,F8.2)
      DO 1 I=1,50
      DO 1 J=1,50
      SVOUT(I,J)=0
      SVPSI(I,J)=0
      SVOMA(I,J)=0
      PSI(I,J)=0
1     OMA(I,J)=0
      NM=0
      F2=1-F1
      C2=1-C1
C     BEGIN LOOP FOR OUTER ITERATIONS
C     SAVE VORTICITY FUNCTION FROM PREVIOUS OUTER ITERATION
23    DO 40 I=1,NPLUS1
      DO 40 J=1,MPLUS1
40    SVOUT(I,J)=OMA(I,J)
      NM=NM+1
      NCOUNT=0
C     BEGIN INNER ITERATION FOR STREAM FUNCTION
C     COMPUTE STREAM FUNCTION FOR INNER REGION
11    DO 2 I=3,NMESH
      DO 2 J=3,MMESH
      SVPSI(I,J)=PSI(I,J)
2     PSI(I,J)=(-.8*PSI(I,J))+.45*(PSI(I,J-1)+PSI(I,J+1)+PSI(I-1,J)+
     1PSI(I+1,J)+H2*OMA(I,J))
C     COMPUTE STREAM FUNCTION ON TOP AND BOTTOM INNER BOUNDARY LINES
      DO 3 I=2,N
      PSI(I,2)=(.25*PSI(I,3))
3     PSI(I,M)=.25*PSI(I,MMESH)+.5*H
C     COMPUTE STREAM FUNCTION ON LEFT AND RIGHT INNER BOUNDARY LINES
      DO 4 I=3,MMESH
```

```
      PSI(2,I)=    (.25*PSI(3,I))
4     PSI(N,I)=    (.25*PSI(N-1,I))
C     TEST STREAM FUNCTION FOR CONVERGENCE
      DO 5 I=3,NMESH
      DO 5 J=3,MMESH
      DIFF=ABSF(SVPSI(I,J)-PSI(I,J))
      IF(DIFF .GT. EPS) GO TO 6
5     CONTINUE
C     RECALCULATE STREAM  FUNCTION USING WEIGHTING
      DO 222 I=3,NMESH
      DO 222 J=3,MMESH
222   PSI(I,J)=F1*SVPSI(I,J)+F2*PSI(I,J)
      DO 114 I=2,NMESH
      IF(PSI(I,M))28,114,114
114    CONTINUE
      GO TO 200
6     NCOUNT=NCOUNT+1
      IF(NCOUNT .GT. 100) GO TO 8
      GO TO 11
C     TEST STREAM FUNCTION FOR DIVERGENCE
8     IF(DIFF .GT. 10) GO TO 28
      PRINT 93
93    FORMAT(1H1,11H PSI VALUES)
      CALL PRNTLST(PSI)
10    FORMAT(10F11.6)
      NCOUNT=0
      GO TO 11
28     PRINT 81
81    FORMAT(13H PSI DIVERGED)
      CALL PRNTLST(PSI)
      CALL PRNTLST(OMA)
      GO TO 699
C     BEGIN INNER ITERATION FOR VORTICITY
200   NCOUNT=0
30     HCONST=C2*(-2./H2)
C     COMPUTE VORTICITY ON BOUNDARY LINES USING WEIGHTING
C     TOP AND BOTTOM BOUNDARY LINES
      DO 12 I=1,NPLUS1
      OMA(I,1)=C1*OMA(I,1)+HCONST*PSI(I,2)
12    OMA(I,M+1)=C1*OMA(I,M+1)+HCONST*(PSI(I,M)-H)
C        LEFT AND RIGHT BOUNDARY LINES
       DO 13 I=2,M
      OMA(1,I)=HCONST*PSI(2,I)+C1*OMA(1,I)
13     OMA(N+1,I)=HCONST*PSI(N,I)+C1*OMA(N+1,I)
96    CONTINUE
C     COMPUTE COEFFICIENTS FOR VORTICITY EQUATIONS
C     COMPLETE ONE SWEEP OF INTERIOR
      DO 14 I=2,N
      DO 14 J=2,M
      A1=PSI(I+1,J)-PSI(I-1,J)
      B1=PSI(I,J+1)-PSI(I,J-1)
      A=ABSF(A1)
      B=ABSF(B1)
      W0=4+(A+B)*(R/2)
       IF(A1.GE. 0)15,16
```

```
15        W2=1+(R/2)*A
          W4=1
          GO TO 17
16        W2=1
          W4=1+A*(R/2)
17        IF(R1.GE. 0)18,19
18        W1=1
          W3=1+B*(R/2)
          GO TO 20
19        W1=1+B*(R/2)
          W3=1
20        SVOMA(I,J)=OMA(I,J)
          IF(ISTOP .EQ. 1)GO TO 305
         OMA(I,J)=((W1/WO)*OMA(I+1,J)+(W2/WO)*OMA(I,J+1)+(W3/WO)*OMA(I-1,J)
        1+(W4/WO)*OMA(I,J-1))*RW+(1-RW)*OMA(I,J)
          GO TO 14
C         CHECK TO SEE IF DIFFERENCE EQUATIONS ARE SATISFIED TO .001
305       DIFF    =((W1/WO)*OMA(I+1,J)+(W2/WO)*OMA(I,J+1)+(W3/WO)*OMA(I-1,J)
        1+(W4/WO)*OMA(I,J-1))-OMA(I,J)
          DIF=ABSF(DIFF)
          IF(DIF .GT. EPS1)282,14
282       PRINT 183,I,J
          GO TO 700
14        CONTINUE
          IF (ISTOP .EQ. 1) GO TO 700
C       TEST    VORTICITY      FOR CONVERGENCE
          DO 21 I=2,N
         DO 21 J=2,M
          DIFF=ABSF(SVOMA(I,J)-OMA(I,J))
          IF(DIFF .GE. EPS) GO TO 22
21       CONTINUE
C         RECALCULATE VORTICITY          USING WEIGHTING
          DO 144 I=2,N
          DO 144 J=2,M
144        OMA(I,J)=C1*SVOMA(I,J)+ C2*OMA(I,J)
          JM=JM+1
C         PRINT OUT EVERY 4 OUTER ITERATES
          IF(JM .EQ. 4)89,59
89        JM=0
          PRINT 79,NM
79        FORMAT(1H1,I2,17H OUTER ITERATIONS)
          PRINT 91
          CALL PRNTLST(PSI)
          PRINT 92
          CALL PRNTLST(OMA)
C       TEST OUTER ITERATIONS FOR CONVERGENCE
59        CONTINUE
          DO 45 I=1,NPLUS1
          DO 45 J=1,MPLUS1
          DIFF=ABSF(SVOUT(I,J)-OMA(I,J))
          IF(DIFF .GT. EPS) GO TO 7
45         CONTINUE
          NZ=0
          MP=8
           PRINT 99,NM
```

```
 90     FORMAT(1H1,22H PROBLEM CONVERGED IN ,I4)
        PRINT 91
 91     FORMAT(1X,11H PSI VALUES)
        CALL PRNTLST(PSI)
        PRINT 92
 92     FORMAT(1H1,14H OMEGA VALUES)
        CALL PRNTLST(OMA)
        EPSI=.001
        RMAX=0
        ISTOP=1
C       CHECK TO SEE IF DIFFERENCE EQUATIONS OR STREAM FUNCTION ARE SATISFIE
C      A TOLERANCE OF .001
        DO 181 II=3,NMESH
        DO 181 JJ=3,MMESH
        RES=ABSF(PSI(II,JJ)-SVPSI(II,JJ))
        IF(RES .GT. RMAX)301,302
 301    RMAX=RES
 302    CONTINUE
        A=-4*PSI(II,JJ)+PSI(II+1,JJ)+PSI(II,JJ+1)+PSI(II-1,JJ)+PSI(II,JJ-
       11)
        B=-H*H*OMA(II,JJ)
        D=A-B
        IF(D .GT. EPS1) GO TO 182
 181     CONTINUE
        GO TO 90
 182    PRINT 183,II,JJ
 183    FORMAT(1H1,41H DIFFERENCE EQU. NOT SATISFIED AT POINT (,I2,1H,,I2
       1,1H))
         GO TO 699
C      TEST OUTER ITERATIONS FOR DIVERGENCE
 7      IF(DIFF .GT. 100)199,23
 22     NCOUNT=NCOUNT+1
        IF(NCOUNT .GT. 300) GO TO 24
        GO TO 90
C      TEST   VORTICITY    FOR DIVERGENCE
 24     IF(DIFF .GT. 10) GO TO 29
        PRINT 94
 94     FORMAT(1H1,14H OMAEGA VALUES)
        CALL PRNTLST(OMA)
        PRINT 91
        CALL PRNTLST(PSI)
 32     FORMAT(10F11.6)
        NCOUNT=0
        GO TO 90
 29     PRINT 82
 82      FORMAT(13H OMA DIVERGED)
        CALL PRNTLST(PSI)
        CALL PRNTLST(OMA)
        GO TO 699
 199    PRINT 189
 189    FORMAT(26H OUTER ITERATIONS DIVERGED)
 700     CONTINUE
        PRINT 303,RMAX
 303    FORMAT(1H1,17H PSI CONVERGED TO,E12.4)
 699    CONTINUE
```