

ON CALCULATING WITH B-SPLINES

II. INTEGRATION

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This paper is a continuation of the paper [1] of the same name by the first author in which it is shown how values of B-splines and their derivatives can be computed by stable algorithms based on recursions involving only convex combinations of nonnegative quantities (cf. also Cox [3]). In this paper we consider integrals of B-splines and of B-spline series. In addition, we derive recursions for the computation of integrals of products of B-splines (of possibly different orders and on possibly different knot sequences). As an application, we consider the numerical computation of the Gram matrix which arises in least squares fitting using B-splines.

§ 1. Introduction

We begin by introducing some basic notation. Let k be a positive integer, and suppose \bar{x} denotes a finite sequence of real numbers

$$(1.1) \quad \dots \leq x_{-2} \leq x_{-1} \leq x_0 \leq x_1 \leq \dots$$

with at most k values equal to each other (i.e., $x_1 < x_{1+k}$ for all 1).

