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Subroutine Package for Calculating with B-Splines

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ABSTRACT

Seven FORTRAN subprograms are presented for dealing with piecewise polynomial functions (of one variable) computationally. The package is built around an algorithm for the stable evaluation of B-splines of arbitrary order. Three examples illustrate what uses one might make of these routines: interpolation by splines of general order k (and not necessarily at the knots), the determination of the derivative of a spline with respect to a knot, and the approximate solution of an ordinary linear differential equation by collocation.

1. REPRESENTATIONS

In this set of subroutines, a piecewise polynomial function is represented either in terms of its local polynomial pieces (pp-repr.) or in terms of its coefficients with respect to the appropriate B-spline basis (B-repr.).

More precisely, the pp-representation for a piecewise polynomial function, $s(t)$, consists of:

The integers K and LXI , giving the order (i.e., $K - 1$ is the degree) and the number of polynomial pieces, respectively;

The one-dimensional array $XI(i)$, $i = 1, \dots, LXI$, giving the break points (in increasing order); and

The two-dimensional array $C(j,i)$, $j = 1, \dots, K$; $i = 1, \dots, LXI$, with

$$C(i,j) = s^{(j-1)}(XI(i)),$$

the various derivatives of s at the various break points.

From this, $s^{(j)}(t)$ is found (in PFVALU) as

$$s^{(j)}(t) = \sum_{r=j}^{K-1} C(r+1,i)(t - XI(i))^{r-j}/(r-j)!$$

where i is such that

$$i = 1 \text{ and } t < XI(2),$$

$$\text{or } 1 < i < LXI \text{ and } XI(i) \leq t < XI(i+1),$$

$$\text{or } i = LXI \text{ and } XI(LXI) \leq t.$$

The B-representation for a piecewise polynomial function, $s(t)$, consists of:

The integers K and N , giving the order (as a spline) and the number of linear parameters for s , respectively;

The one-dimensional array $T(i)$, $i = 1, \dots, N + K$, containing the joints (possibly partially coincident) in increasing order; and

The one-dimensional array $A(i)$, $i = 1, \dots, N$, containing the coefficients with respect to the B-spline basis on T .

